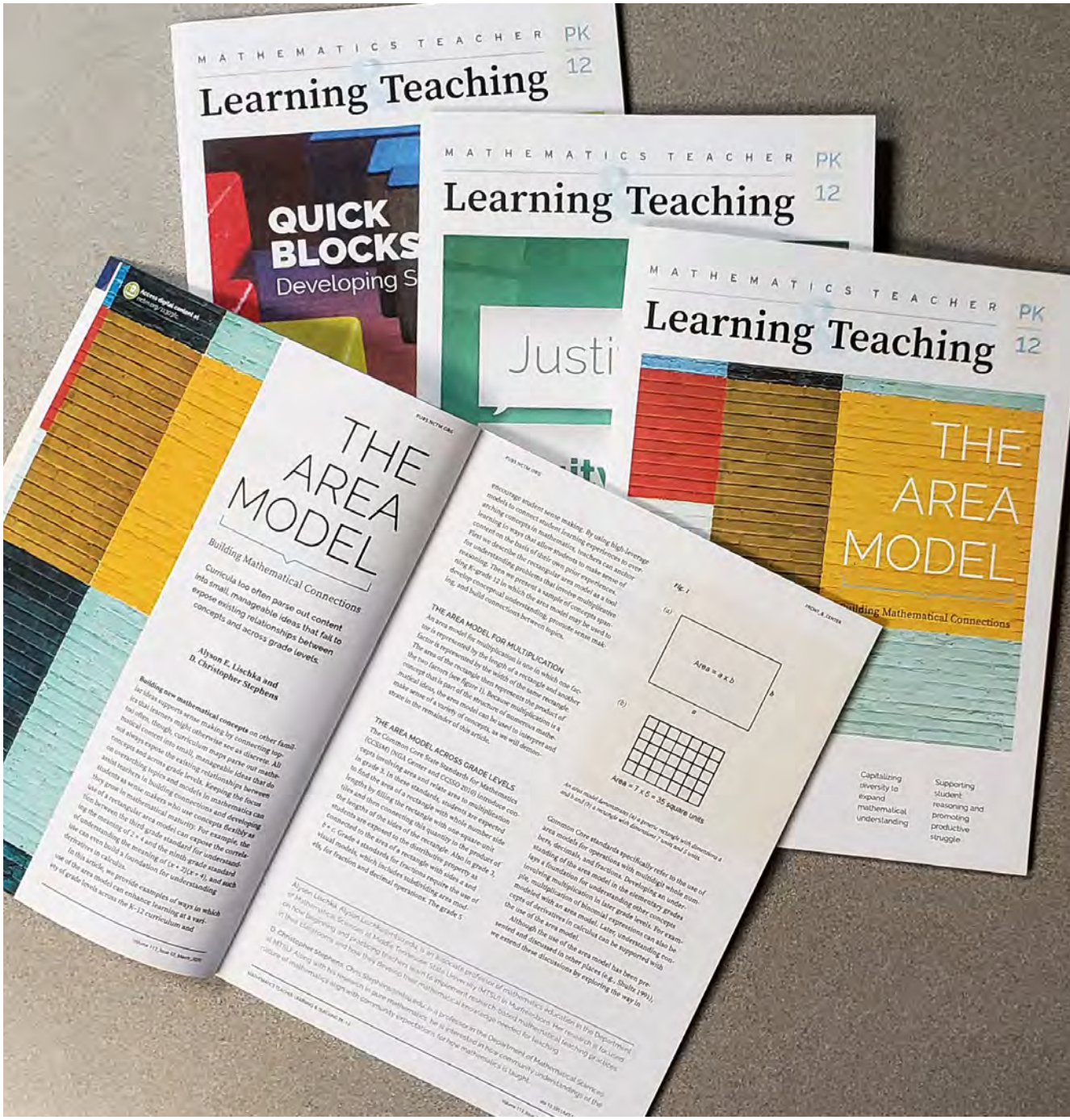
MATHEMATICS TEACHER

Learning & Teaching

PK
12

Author TOOLKIT



NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Welcome!

Welcome, *MTLT* author!

Thank you for your interest in *MTLT*, NCTM's new digital-first practitioner journal offering useful and classroom-ready content for the elementary, middle, and high school grade bands as well as critical information on subjects across the PK–12 spectrum. If you're a classroom teacher of PK-12 mathematics, we invite you to submit to *MTLT*. By publishing in the journal, you will have a long-term impact on other teachers and their students through your teaching success stories. *MTLT* is practitioner journal by and for PK-12 mathematics teachers!

We realize that classroom teachers have little time to navigate the complexities of creating, submitting, and revising a journal manuscript. Our goal with this Author Toolkit is to explain the manuscript submission process step by step for interested authors. We do this by defining what we mean by **digital first**, describing manuscript types that the journal publishes, providing instructions on preparing and submitting a manuscript, and, finally, detailing the steps of peer review and the postacceptance workflow.

Thank you again for your interest in *MTLT*. We are looking forward to receiving your submission and hope that you are excited about NTCM's digital-first journal.

Sincerely,
Angela Barlow, PhD
Editor-in-Chief, *MTLT*

Stephanie Dean
Managing Editor, *MTLT*

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WHAT IS *MTLT*?

Mathematics Teacher: Learning and Teaching PK–12 (*MTLT*) is NCTM's exciting new digital-first journal with a unique perspective—your view as a practicing teacher of mathematics. The journal spans PK–12 and continues to provide grade-band–specific articles as well.

Manuscripts in *MTLT* can vary in type and length. A **Front-and-Center** article has a message that appeals to *MTLT* readers working with students across all grades, PK–12. This can be achieved in many ways, including, but not limited to, writing on an issue of importance to all schools, providing relevant examples for different grade bands, or reframing the topic for each grade band in different sections of the article. A **Feature** article emphasizes one grade or grade band. Evidence of classroom implementation is expected when appropriate. A **Focus** article shares a single, developed idea to be quickly read by the busy teacher. Teaching strategies, learning technologies, or tasks used for a lesson are but a few of the many possibilities. You may also write for a **Department**—these are articles that often originate from your daily interactions with students.

WHAT IS DIGITAL FIRST?

A digital-first journal takes advantage of 21st-century technology to go beyond the printed word. For example, instead of a static figure in a printed article, *MTLT* would prefer to include an embedded video that puts the reader inside the classroom where student–student and student–teacher interactions can be observed. Audio, apps, or interactive whiteboard files are just a few of the many opportunities that the new journal holds for authors and readers. Authors are encouraged to include such digital components as a short video clip, GeoGebra activity, Desmos exercise, Livescribe™ file, SMART Board™ file, or other form of multimedia to enhance their submission (however, such use does not imply NCTM endorsement of any product or developer).

In appendix 1, we provide examples of digital assets in *MTLT* articles.

WRITE FOR *MTLT*

MTLT features three main types of articles:

- **Front-and-Center Article—Grade Band PK–12**

  A Front-and-Center article has a message that appeals to *MTLT* readers working with students across all grades, PK–12. This can be achieved in many ways, including, but not limited to, writing on an issue of importance to all schools (providing relevant examples for different grade bands) or reframing the topic for each grade band in different sections of the article. Submissions should not exceed 5,000 words. See a sample in appendix 2.

- **Feature Article—Grade Bands PK–2, 3–5, 6–8, 9–12**

  A Feature article emphasizes one grade or grade band. Evidence of classroom implementation is expected when appropriate. Submissions should not exceed 3,500 words. See a sample in appendix 3.

- **Focus Article—Grade Bands PK–2, 3–5, 6–8, 9–12**

  Focus articles share a single developed idea to be quickly read by the busy teacher. Teaching strategies, learning technologies, or tasks used for a lesson are but a few of the many possibilities. The evidence of classroom implementation will not be the main message of the article. Submissions should not exceed 1,500 words. See a sample in appendix 4.

MTLT offers other article types as well:

- **Ear to the Ground** (Department)

  The Ear to the Ground department highlights voices working in various corners of the mathematics education world. Readers hear from book authors, conference presenters, bloggers, and others and get accompanying digital enhancements to keep up to date on current happenings. Contributors are invited by the editors. Submissions should not exceed 1,000 words. See a sample in appendix 5.

- **Editorial**

  Editorials are short articles that comment on issues of significance to mathematical thinking and learning. Authors should present the issue in a clear and constructive way and then add their critique or suggestion(s) on improving or expanding the idea. Alternatively, the author may present both sides of an issue and leave it up to readers to decide their personal point of view. Successful editorials will leave readers wanting to know more or will help them engage in the topic at hand. Authors are encouraged to include in their submission the invitation to continue the discussion on myNCTM (NCTM's discussion board) or Twitter (copy @nctm) and Facebook (tag @NCTM). Subject matter could include such topics as curriculum, pedagogy, assessment, mathematics teacher education, educational

philosophy, research implementation, structure of the educational system, special teacher needs, or special student needs.

- **For the Love of Mathematics** (Department)

  This reader-driven department is intended for the teacher rather than for direct use with students. Items sent to the editor may include a photograph combined with a funny or engaging mathematical question; an original cartoon that is humorous or reflective; a puzzle; Math Circle prompts; a poem or vignette; original artwork; or general celebratory items, such as Pi Day or Metric Week. Submission of student work within these guidelines is also welcome. How do you show your love of mathematics? Submissions should not exceed 150 words. See a sample in appendix 6.

- **Growing Problem Solvers** (Department)

  Send in a rich task or tasks that can potentially span four points along the PK–12 learning trajectory (PK–2, grades 3–5, grades 6–8, and grades 9–12) and can elicit higher-order thinking. A "teacher" page introduces the math and potential instructional strategies. The subsequent text then applies the task to each of the four grade bands. If you have used a rich task in your classroom and have had to extend it, contribute to this discussion. Submissions should not exceed 1,500 words ("Teacher" page = 500 words; each grade-band task = 250 words). See a sample in appendix 7.

- **Letter to the Editor**

  A Letter to the Editor should be a brief response to an article, discussing only issues directly relevant to the original article's content. The response may be supportive or critical in nature. One example would be a letter that offers an alternative perspective or counterpoint to an article's main focus. Readers are encouraged to direct general comments ("I really enjoyed the article by _____") or personal reflections ("I used activity mentioned in _____ in my classroom, and the children loved it!") on *MTLT* articles to myNCTM, the NCTM discussion board. Additionally, the online journal allows users to post about a specific article on Twitter and Facebook. When tweeting, be sure to tag @NCTM and use these hashtags: #MTLTPK12, #MTBoS, #iteachmath, #T2T, #math, #mathed. If posting on Facebook, tag @NCTM.

- **Problems to Ponder** (Department)

  Individuals are encouraged to submit a problem or a collection of problems by email to mtlt@nctm.org. If published, authors of problems will be acknowledged. See a sample in appendix 8.

- **Teaching Is a Journey!**

  This department provides a space for PK–12 teachers of mathematics to connect with other teachers of mathematics through their stories that lend personal and professional support. Submissions should not exceed 1,600 words.

PREPARING YOUR MANUSCRIPT

Helpful Tools for Manuscript Preparation

- Digital Asset Types and Examples (appendix 1)
- *MTLT* FAQs (appendix 9) for a list of common author questions
- Digital Assets and Figures: Instructions and Technical Specs (appendix 10)
- Figure/Multimedia Permission Form—Adult Participants (appendix 11)
- Figure/Multimedia Permission Form for Non-Adult Participants (those younger than 18 years old; requires parental completion and signature) (appendix 12)
- You've Got Stories! Now It's Time to Write – Webinar (length, ~50 minutes)

Ethical Guidelines

In preparing your manuscript, ensure that your materials are not under consideration for publication elsewhere or have not been published elsewhere.

Permissions/Consent from Individuals Shown in Videos or Figures

Adults appearing in videos or figures must complete a **Video Permission** (see appendix 11) form allowing NCTM to publish their image. **IMPORTANT**: For participants younger than 18 years, we require a completed **NCTM Release Form for Use of Student's Image or Written Work** (appendix 12) from each student's parent/legal guardian. The forms can be uploaded individually or combined into one PDF document for submission to ScholarOne.

Permissions for Student Written Work

Parental consent is also required for distinctive original written work created by students. For each such case, you will upload a completed **NCTM Release Form for Use of Student's Image or Written Work** (appendix 12) signed by the student's parent or legal guardian. Alternatively, you may combine the signed forms into one file for submission.

Reprinting or Adapting Materials from Other Sources

Authors are responsible for obtaining permissions for any copyrighted material included in their article. Copyrighted material typically includes adapted or reprinted tables, appendixes, or figures from existing publications or websites. Although permission-to-reprint documentation is required only if a manuscript is accepted, the process of securing permission can take time. Therefore, we encourage authors to contact the publisher of the original material early in the peer review process.

Word Count

The manuscript text, tables, and references are included in the total word count of each submission. Word counts by article type are defined in the WRITE FOR MTLT section.

Blinding

MTLT is a double-blind journal, meaning authors do not know the identity of reviewers and reviewers do not know the identity of authors. Associate editors and the editor-in-chief are *not* blinded to author names.

To preserve anonymity, author(s) names may appear *only* in an author cover letter that is separate from the manuscript file. Please ensure no author identification appears within the manuscript. Do not reference your own work or your school in a way that compromises the blind review process. For example:

Incorrect: We (Smith and Jones 2017) tested students' knowledge after implementation . . .

Correct: Author (2017) tested students' knowledge after implementation . . .

Use pseudonyms for students' or colleagues' names and blind project or grant information, acknowledgments, and conference presentations appearing in a manuscript. Links to personal or institutional websites should also be blinded. Reference list entries should be formatted as follows: Author (year); for example, Author (2017).

General Writing Guidelines

- Order your manuscript text file as follows:
  - Anonymized title page
  - Manuscript body (include figures [with figure legends], video still images, and tables within the manuscript text, near their first mention)
  - Reference list
  - Appendixes
- Refer to the *Chicago Manual of Style*, 17th ed. (Chicago: University of Chicago Press, 2017) and the June 2020 NCTM Style Guide for complete style guidelines.
- Set text in Times New Roman, 12 point, double-spaced throughout, including quoted matter, lists, tables, notes, and references.
- Use formatted headings (up to four levels) to help organize your article for the reader.
- Please proofread and spell-check your manuscript *before* submission. Review it for grammar, completeness, mathematical correctness, and accuracy of references.
- Provide tables using the Microsoft Word table feature. Do not import an image-based table.
- Ensure all tables, figures, videos, or other multimedia material are mentioned in text and, on first mention, they are numerically ordered. Embed figures, tables and video still images within the text, near their first mention.
- *MTLT* prefers that footnotes be eliminated. Please integrate them parenthetically into the main text instead.
- Ensure that all in-text citations are included in the reference list, and, likewise, that all reference list entries are cited in text.
- For quotations in text, include the quotation's page number(s) in the in-text citation.
- Use extended dialogue and direct quotes sparingly, incorporating the key ideas of conversations into the text wherever possible.

References

- View *MTLT* Reference Style Examples (appendix 13)

MTLT uses the *Chicago Manual of Style* (17th ed.) for in-text citations and reference lists.

Figures and Multimedia Files

- When determining whether to add a figure, determine first whether the figure will add educational value to the text. For example, photographs of students working at their desks or the whiteboard often do not add appreciably to the content of an article. Include figures that advance the understanding of the article's main point(s).
- Information contained in tables, figures, videos, and digital assets should not be repeated in text. Rather, the text should be written under the assumption that the reader has read or viewed these items.
- During peer review, figures must be legible but do not have to be provided as high-resolution files. Should your manuscript be accepted, however, you will provide high-resolution images, which will be used for production. The Multimedia/Figure Guidelines and Technical Specifications (appendix 10) provide instructions on required specifications and image quality for final figure files.
- Within the manuscript text file, figure legends—labeled by figure number—should be placed within the manuscript body, beneath the figures they describe.
- Figures should be called out, on first mention, in numerical order within the manuscript text. For example, figure 2 should not be cited in text before figure 1.

Mathematical Characters and Equations

- All variables should be set in italics.
- Numbers, parentheses, and mathematical operators should not be set in italics.
- Points, as in "segment *AB*," should be italicized, and labels in geometric figures must also be italicized.
- Do not use MS Word Equation Editor for mathematical expressions.
- Most of the mathematics in *MTLT* can be written in regular Microsoft Word characters and symbols, using italics, superscripts, and subscripts as necessary. For example:

$(a + b)^2 = a^2 + 2ab + b^2$ $\qquad$ $k(x) = 4 - y$

$|2 - x| \leq 6$ $\qquad$ $A_1, A_2, \ldots A_n$

- Fractions should be written in Microsoft Word without any formatting or stacking; they will be formatted during layout as needed. For example:

$1/2x + 3/4y = 4$ $\qquad$ $4\ 1/2 - 1\ 1/8 = 3\ 3/8$

- Use MathType sparingly for expressions and equations that cannot be typed using the keyboard, such as in the examples below.

- Since most expressions set in MathType will require their own line in the journal, that should be a guiding principle for authors; **if the expression is not important enough to be broken out on its own line, then do not use MathType**.
- For more complex equations (e.g., the two equations below) that require MathType, authors should be sure to include versions within their manuscript that show how the equations should be arranged. A variety of programs are available to create mathematical equations, and authors should indicate in a manuscript note how their equations were created.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 1$$

$$\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

SUBMITTING YOUR MANUSCRIPT TO THE SCHOLARONE PEER REVIEW SYSTEM

Helpful Tool for the ScholarOne System

- How to Submit to ScholarOne–A guided PowerPoint® provides an overview of the submission site.

All submissions for *MTLT* are handled through the ScholarOne peer review system, with the exception of the **Problems to Ponder** department (which should be emailed to mtlt@nctm.org).

General Guidelines for the ScholarOne System

- You will be asked to assign copyright of your article to NCTM as part of the submission process. Please note that the copyright agreement is voided if your manuscript is not accepted for publication.
- If you have coauthors, you will enter each coauthor's name, their affiliation, address, and email into the system. Gathering this information in advance of submission can be helpful.
- For peer review, you will include a manuscript text file (MS Word) consisting of an anonymized title page; the manuscript body that includes embedded figures and tables near their respective first mentions in text; a reference list; and appendixes, in that order.
- If your submission contains figures or tables, you will embed them within the manuscript text file.
- If your submission contains media file(s) (e.g., video, GeoGebra, etc.), consider the following:
  - If possible, the media source file(s) should be uploaded to the peer review system, rather than adding a link within the text of your manuscript to an external website. For example, instead of providing a link in your manuscript to a self-authored YouTube video, we prefer that you upload the video source file to the ScholarOne

system (however, if you are not the author of the video, an in-text link is appropriate). Please note that the ScholarOne submission system limits file sizes to 100 MB. If you have large videos, narrated PowerPoints, or other multimedia files, please contact the editorial office for alternative upload instructions.

A Note about Consent and/or Permission

Parental consent forms for students under 18 years who appear in figures or videos are not required at submission but will become required should your paper be accepted. It is best to gather these consent forms before photographing or recording students, however, to ensure you will be able to use the image(s) or footage you obtain. The parental consent form is available in appendix 12.

THE PEER REVIEW PROCESS

Once submitted to ScholarOne, your manuscript will be assigned to an associate editor (AE) and will undergo double-blind peer review by three reviewers. Reviewers are allowed 21 days to return their assessments.

After all reviews have been received, the AE will also read your manuscript and then integrate his or her comments with those of the reviewers to arrive at a recommended decision about the suitability of the manuscript for publication in *MTLT*.

After the AE has submitted a recommendation, your manuscript moves into the editor-in-chief's (EIC's) queue for final review. Once the EIC has entered his or her decision, the ScholarOne system will email you the decision letter. If a revision is requested, the AE will summarize the critical points from the reviewers' comments that you will need to focus on when revising your manuscript.

***MTLT* Decision Types**

MTLT offers six decision types: (1) Accept, (2) Conditional Accept, (3) Minor Revision, (4) Major Revision, (5) Reject & Resubmit, and (6) Reject. It should be noted that neither a major nor minor revision decision represents a commitment to accept. If you receive a revision decision, please attend carefully to all feedback in order to improve the chances of having your article accepted.

Accept: Your paper is accepted as is, without needing further edits. Editorial staff will contact you within 2 weeks after acceptance with a request for your final files.

Conditional Accept: There are a few minor or cosmetic changes that are needed before your paper can be accepted.

Minor Revision: There are some (typically) nonsubstantive issues that need resolution or further clarification before a final decision can be rendered.

Major Revision: Your paper has merit but there are significant issues to address before a final decision can be considered.

Reject with Encouragement to Resubmit: The concepts or activities in your paper are of interest, but the framing of your argument might need reworking, more classroom activity may be needed to support your conclusions, or there may be other critical issues that need to be addressed before publication can be considered. Reject & Resubmit decisions essentially provide you an opportunity to "revise" your paper under a new manuscript number.

Reject: Unfortunately, due to space and scope, reject decisions are not uncommon. It is important to remember that a reject decision does not mean your paper is without merit or a lacking a path toward publication. A reject decision is often not a reflection of quality of the paper, but rather, for example, a lack of fit within the journal's mission and scope, or the impression during review that the conclusions or assertions were not documented through descriptions of classroom implementation. At *MTLT,* we hope authors will view a reject decision as a challenge to improve their article, for potential new submission to *MTLT* or to another publication.

MTLT will consider previously rejected manuscripts when authors take into account the issues raised in peer review and create a new manuscript that reflects that feedback. If you submit a new manuscript that dovetails from a previous reject decision, you should cite the previous manuscript in your cover letter and explain how the new manuscript addresses critical feedback from the review process. You will upload the manuscript as a new submission in ScholarOne.

AFTER ACCEPTANCE

Should your manuscript be accepted, the ScholarOne system will move your manuscript into the "**Manuscripts Accepted for First Look**" area of your Author Dashboard soon thereafter. Approximately one or two weeks after acceptance, you will receive an email from the editorial office that provides instructions for uploading the files and forms that are needed for production. Please try to upload your final files within one week.

Production

Your accepted manuscript will be copyedited in accordance to NCTM style guidelines. After copyediting, your manuscript will be composed into page proofs that represent how the published article will appear in print. You will receive a PDF copy of these proofs by email with instructions on returning your changes to the production vendor. Please be sure to adhere to the requested 48-hour turnaround time for review of these proofs.

Appendixes

Appendix 1. Digital Asset Possibilities for *MTLT*

Background

Often, when we mention digital assets, we immediately think of classroom videos. Classroom videos can be very useful, but there are many other possibilities for representing classroom instruction and other digital assets in *MTLT*.

Below are some types of digital assets that could be included in an *MTLT* article. We also provide sample article from *MTLT* showcasing each media type.

Examples of digital assets

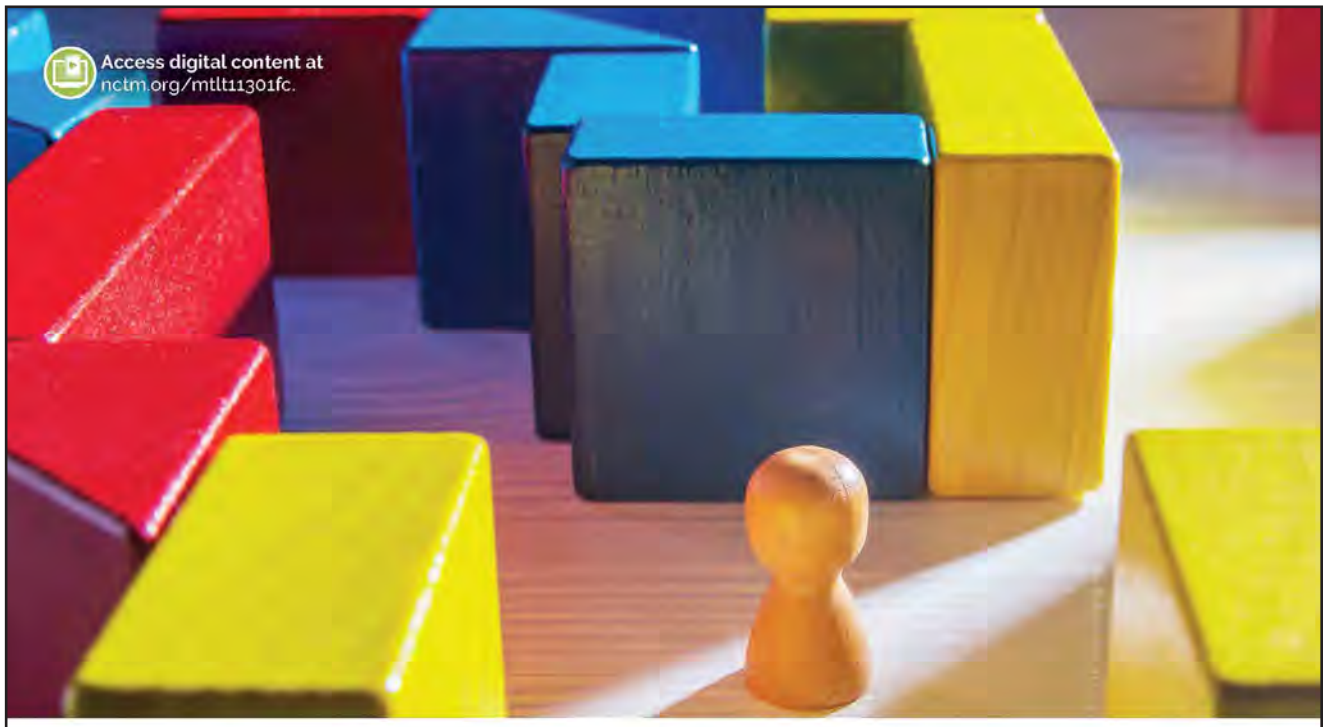
Consider using a digital mathematics environment (DME), video(s) of students working and discussing, screencast/Livescribe student voices without faces while working and discussing, virtual manipulatives, myNCTM Discussion Board prompts, video and/or audio to pose a task, or innovative and dynamic navigation/organization.

Digital Asset	Description of Example(s)	Example Article(s)
Video Clip	Video modeling force Video with students engaged with activity in a computer lab Integrate YouTube video with gravity applied in space to make a realistic connection.	**Exploring the Mathematics of Gravity** Authors: Debasmita Basu, Nicole Panorkou, Michelle Zhu, Pankaj Lal, Bharath K. Samanthula Vol. 113, No. 1, January 2020 doi: https://doi.org/10.5951/MTLT.2019.0130
Audio File	Used as part of an activity	**Modeling a Bouncing Ball with Exponential Functions and Infinite Series** Author: Tim Erickson Vol 113, No. 3, March 2020 doi: https://doi.org/10.5951/MTLT.2019.0042
Desmos Activity	Experiment with equations of sinusoidal graphs	**Envelope Curves Unify Sinusoidal Graphing** Authors: Christopher Harrow, Nurfatimah Merchant Vol. 113, No. 4, April 2020 doi: https://doi.org/10.5951/MTLT.2019.0129

Appendix 1 (continued)

Digital Asset	Description of Example(s)	Example Article(s)
myNCTM **Discussion Board Prompts**	Engage with readers about your article by directing them to the myNCTM Discussion Board for continued conversation.	**What's in a Name? Language Use as a Mirror into Your Teaching Practice** Authors: Tracy E. Dobie, Miriam Gamoran Sherin Vol. 113, No. 5, May 2020 doi: https://doi.org/10.5951/MTLT.2019.0296
Interactive Simulation and Video	Use of Projectile Motion allows students to work with several variables that affect the launch of a cannonball, including changing the angle of launch, the height of the cannon, the speed of the ball, the mass of the cannonball, and the distance of the target on the ground from the cannon.	**Let's Hit the Target** Authors: Manouchehri Azita, Ozturk Ayse, Sanjari Azin Vol. 113, No. 5, May 2020 doi: https://doi.org/10.5951/MTLT.2019.0022
Virtual Manipulative or Game	Use of Mastermind, an online game in which the transition toward conditional reasoning is seen as a learning progression and not a cognitive leap.	**Conditional Reasoning Online with Mastermind** Authors: Sean P. Yee, George J. Roy, LuAnn Graul Vol. 113, No. 5, May 2020 doi: https://doi.org/10.5951/MTLT.2019.0093
Livescribe Pen Files (an Alternative to Video)	Use of Livescribe to demonstrate students' tendency to rely on familiar procedures and ideas instead of taking time to think about and analyze a problem situation.	**Addressing the Hammer-and-Nail Phenomenon** Author: Kien H. Lim Vol. 113, No. 6, June 2020 doi: https://doi.org/10.5951/MTLT.2019.0018

Appendix 2. Sample Front-and-Center Article



QUICK BLOCKS:
Developing Spatial Sense

Videos and classroom dialogue show the importance of giving K–12 students opportunities to engage in spatial reasoning and number sense activities.

Gabriel Matney, Julia Porcella, and Shannon Gladieux

Have you ever noticed that some students seem to be able to visualize three-dimensional (3D) objects or situations within problems better than others? Through our teaching experiences, we have observed that students' spatial sense improved as they worked with 3D manipulatives during problem solving. Encouraged by these teaching experiences, we set out to create a short classroom routine that our students would find challenging and enjoyable and that would allow opportunities to develop spatial reasoning as well as number sense. This article explores the importance of giving students time to develop mental imagery in mathematics, especially 3D imagery involving cubes. We give a description of the development process we conducted to design the Quick Block images, and we share our experiences of engaging students in the Quick Blocks activity. We also give free access to our electronic files to be used by teachers with their students at all levels.

This article is for demonstration purposes only and may not be re-used without permission.

Appendix 2 (continued)

THE IMPORTANCE OF SPATIAL SENSE

Spatial sense has been an important area of interest for mathematics educators across several decades. In 1989, NCTM's *Curriculum and Evaluation Standards for School Mathematics* put spatial understanding in clear view for consideration by all mathematics teachers by stating that "spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world" (p. 48). In 1990, *The Arithmetic Teacher* journal published a Focus Issue on spatial sense, stating that "the development of spatial sense is vital for learners of all ages" (Shaw, p. 5). The ability to visualize and imagine mathematical ideas through mental imagery has come to be understood as not only something mathematicians do (Sfard 1994) but also an important part of mathematical reasoning (Brown and Wheatley 1997). As children engage in mathematical activity, they use imagery to make sense of significant mathematics (Reynolds 1993). The majority of preschool students use mental models to solve arithmetic problems, and this requires the development and use of their visual-spatial working memory (Rasmussen and Bisanz 2005). Furthermore, the development of students' spatial sense is important to their mathematics success (Battista 2001; Lee, Lee, and Collins 2010; Reynolds and Wheatley 1997; Thompson 2016).

In our teaching practice, we have found that tasks involving spatial reasoning afford students of different perspectives and strengths to join the class's mathematical discussions. We consider such access to mathematics an important part of our equitable mathematics teaching practice (NCTM 2014), so when justifiable, we attend to incorporating task elements involving spatial reasoning. Moreover, students' spatial skills are not fixed but rather can grow as they engage in tasks involving spatial reasoning (Uttal et al. 2013). It is therefore important for mathematics teachers to consider tasks that nurture students' spatial sense.

We have noticed that asking our students to reflect on imagery builds their enjoyment during learning and has allowed them to make deeper connections across mathematical ideas (Putri 2019). During the planning of our mathematics lessons, we consider ways in which spatial reasoning can support student engagement in other mathematical topics. One area of importance for us is connecting spatial sense with number sense. Children who develop stronger spatial sense have been found to be more successful in their number sense (Gunderson et al. 2012). Recognizing that children's spatial sense can grow and that it helps strengthen other mathematical areas inspired our hopes and designs of the Quick Blocks activity.

DEVELOPMENT OF QUICK BLOCKS

Our ideas for the development of Quick Blocks draw heavily from Wheatley's *Quick Draw* (2007) and number sense ideas found in Wheatley and Reynolds' *Coming to Know Number* (2010). Quick Blocks was built on the basis of three other classroom routines: Quick Draw, Quick Images, and Quick Build.

Classroom Routines for Spatial and Number Sense

As teachers, we have long found benefit in the use of Quick Draw as a routine to develop our students' spatial sense. During a Quick Draw episode, the teacher briefly shows students an image (see figure 1) containing arrangements or pieces of geometric figures and then hides it. Students then draw what they remember seeing. The image is briefly shown a second time, and some students choose to redraw it more precisely or finish their original drawing. Following this, the

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Julia Porcella, jporcel@bgsu.edu, is a ninth-grade teacher of mathematics in the Cleveland Metropolitan School District in Cleveland, Ohio. She is interested in the Standards for Mathematical Practice, lesson study, and student engagement in problem solving.

Shannon Gladieux, shannon_gladieux@olsd.us, is an educator at Olentangy Schools in Lewis Center, Ohio. She is interested in mathematical thinking routines, the Standards for Mathematical Practice, and engaging students in transdisciplinary, problem-based learning.

doi:10.5951/MTLT.2019.0049

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Appendix 2 (continued)

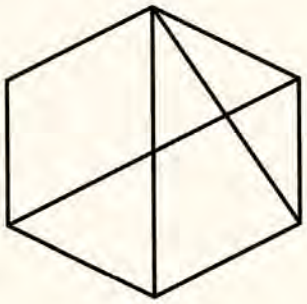
Video 1 Experiencing a Quick Draw Session

Quick Draw

Image Created by Gabriel Matney

Watch the full video online

Fig. 1



Example of a Quick Draw image to engage students in spatial reasoning

students and the teacher converse as a class about what they saw and how they drew it. We have found that this activity opens our students' mathematical minds at the beginning of class. It also improves their ability to sketch and draw, develops their spatial sense, and awakens their noticing of mathematical relationships among geometric figures. To engage in a Quick Draw experience, gather a piece of paper and a writing utensil while watching video 1.

Video 2 offers a glimpse into Shannon Gladieux's fourth-grade classroom as students engage in a Quick Draw episode. The video, which is less than two minutes long, shows students verbalizing different views of the mathematical objects and exploring some academic language involving the geometry seen in the image.

Another type of classroom routine that we use, called Quick Images, promotes subitizing, the ability to know instantaneously how many objects are there (Clements 1999, p. 402). Subitizing is fundamental to the development of other components of number sense, such as unitizing, counting strategies, composing, and decomposing. Quick Images is similar to Quick Draw in that images of dots in various arrangements and quantities (see figure 2) are shown only briefly to students before the image is taken away from view. Students are not shown the image long enough to be able to count one by one. After the image is taken away, students are challenged to consider how many dots

Video 2 Shannon Gladieux Engages Students in Quick Draw

Watch the full video online

Fig. 2

Example of a Quick Image given to students to develop number sense

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Appendix 2 (continued)

Video 3 Experiencing a Quick Image Session

Quick Images

Image Created by Gabriel Matney

they saw and explain how they determined the number of dots. Students see and form ideas about the number of dots in different ways, so as a whole class, students share their perspectives on efficient ways to compose and decompose the number of dots seen.

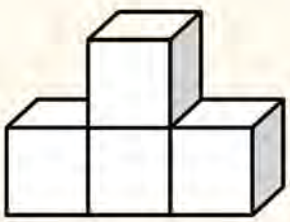
In video 3, we share how Quick Images can differ in arrangement and color, and we explore the reasons teachers might have for showing a particular image. Video 3 also provides an example engagement in Quick Images for those who would like to better understand what it means to instantiate this routine in the classroom.

Quick Draw and Quick Images engage students in important mental mathematics experiences. We continually seek to give our students a variety of classroom routines that allow for productive struggle (NCTM 2014). Having seen that our students' spatial sense benefited from kinesthetic engagement with manipulatives during problem solving, we sought a task that would promote mental mathematics but also would allow for kinesthetic construction. We found the groundwork for such a task in Wheatley and Reynolds' *Coming to Know Number* (2010), in which they briefly describe and share six examples of a task named Quick Build. Similar to the previous classroom routines, students are briefly shown images of 3D cubes (see figure 3) and then asked to construct the image with manipulative blocks that connect. Students discuss what they saw and explain how they built what they saw. We leverage each of these routines to help foster norms for mathematical discourse and for promoting students' construction of viable arguments and ability to critique the reasoning of others (NGA Center and CCSSO 2010). We used our knowledge and experience with these three classroom routines to design the Quick Blocks images.

Designing Images to Connect Spatial and Number Sense

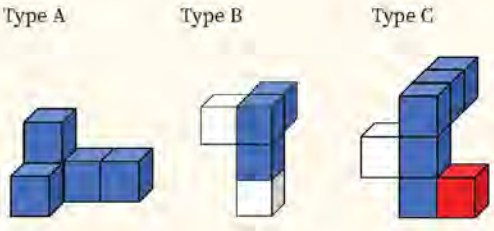
We set out to build on the image templates from Quick Build to connect the development of spatial sense with the subitizing experiences of Quick Images. To do this, we began to create our own 3D cubic images, incorporating different colors, shapes, and orientations. We created images of three different types (see figure 4). The types represent the use of color in the image with Types A, B, and C representing one, two, or three

Fig. 3



Example of a Quick Build image

Fig. 4



Quick Block Type A, Type B, and Type C are differentiated by the number of colors in the image.

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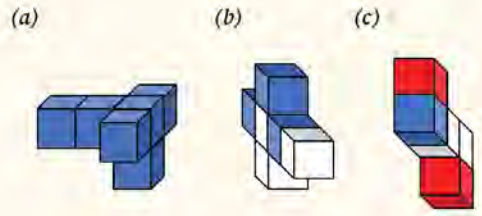
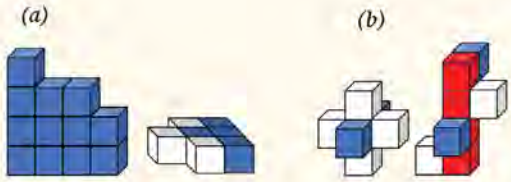
Fig. 5



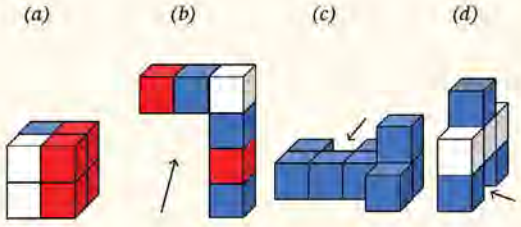
Examples of the types of Quick Block viewing orientations: (a) from above-right, (b) from above-left, and (c) from below-right

Fig. 6



Examples of the types of (a) 2D organization and (b) 3D organization found in Quick Blocks

Fig. 7



Examples of the different types of negative space demarcations in Quick Blocks: (a) no negative space, (b) negative space demarcated by two sides, and (c, d) negative space demarcated by three sides

colors, respectively. To challenge our students' sense of visual orientation, we designed images from different points of view (see figure 5). Next, we wanted our students to experience images in which the cubes might be said to form a single plane, which we refer to as 2D organization, versus situations in which the structure of the cubes depicts 3D organization (see figure 6). Our last consideration was the creation of figures that demarcated negative space in different ways. Figure 7a does not demarcate any negative space; the cubes are all tightly packed. Figure 7b demarcates a negative space with two sides, whereas figures 7c and 7d both demarcate a negative space with three sides.

When engaging students in Quick Blocks, we follow the convention established by Wheatley (2007) whereby students will have at least two opportunities to view the image. The convention is also seen in the videos accompanying this article. The teacher's cue for students to look up at the image is to begin gently counting down—three, two, one—and then briefly reveal the image before concealing it. Students then begin their attempts to physically construct what they saw. As students construct, the teacher observes, noting the ways students are constructing various images and whether the construction matches the image. The teacher then gently counts down a second time—three, two, one—and again briefly shows the image before concealing it. After the second look, students often complete or modify their construction, twisting and lifting it to match the orientation of the image as they seek to ensure the correctness of their construction. The class discusses what students saw initially and how they went about reconstructing the image with the blocks. The teacher asks students to share different observations and to consider the extent to which the observation was helpful. Students discuss what they saw and explain differences in their construction. The teacher reveals the image and allows students to inspect their work. Finally, students suggest new ideas, including efficient and effective strategies to view and subitize the image for successful reconstruction. Teachers can experience a Quick Blocks task by gathering red, white, and blue blocks and watching video 4. Videos 5 and 6 show Julia Porcella's ninth graders and Gladieux's fourth-grade students working on Quick Blocks tasks.

The design of each image incorporates the elements of shape, pattern, and color so that students have a variety of spatial pathways to envision the number of blocks and their orientation to one another. All of our developed Quick Block images can be seen in the Quick Block images file accompanying this article (see the appendix, available as

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Appendix 2 (continued)

an online supplemental file). As videos 5 and 6 and the student scenarios that follow show, students connect different spatial and numeric elements to successfully reconstruct the blocks physically. When teachers provide students with multiple pathways to visualize geometric structures, they increase accessibility for all students (NCTM 2014). In our classrooms, we also seek to nurture a norm of finding more than one way to justify a solution or our thinking about a problem. Such tasks as Quick Blocks—that have multiple pathways through which a student might be successful—open opportunities for students to engage in this norm and to creatively explore the many possible perspectives before any prompting by the teacher.

As students engage in the Quick Blocks task, we want them to be both challenged and successful. Allowing students two opportunities to construct the cubic image is challenging because students naturally want to complete it, if possible, after the first viewing. However, if they cannot complete it or are not yet secure in their result, students know they have a second viewing opportunity. This provides for an enjoyable experience in productive struggle because students have the time for self-correction, self-monitoring, and self-regulation of improvement (NCTM 2014). We noticed that some images were far more likely for students to get correct on the first attempt. We ran trials of each Quick Block image with several classes of fourth-grade students to determine which ones were more difficult. To assist other teachers in knowing which images tended to be more difficult, we organized them into levels: Level 1 images are those most students get correct on the first try; levels 2 and 3 indicate increased difficulty for students. Level 1 images nearly all contain a 2D organization and a small number of cubes. Level 2 images are mostly 3D organizations, contain more cubes than level 1, and have some images demarcating negative space. Level 3 images contain more complex 3D organizations, often contain more cubes than level 1 or 2 images, and some level 3 images demarcate more than one negative space. Each of the three levels contains images consisting of blocks with one, two, and three colors and orientations.

Quick Blocks activities add a kinesthetic element that is not present in Quick Draw and Quick Images tasks. As students process the image, they must mentally represent it and then physically construct it. During the process of construction, students notice that the orientation of the blocks, as they lie on the desk, does not exactly match the orientation of the images. Students must manipulate the blocks, through physical rotation and

Video 4 Julia Porcella's Ninth Graders Engage in Quick Blocks

Watch the full video online

Video 5 Quick Blocks Session Explanation

Watch the full video online

Video 6 Fourth Graders Engage with Two Different Images

Watch the full video online

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Appendix 2 (continued)

Table 1 Modifications When Enacting Quick Blocks Routines

Grade Band	Top 10 Favorites	Modifications
PK–2	AL1-1 AL1-2 AL1-3 AL1-5 AL1-9 AL2-3 BL1-1 BL1-2 CL1-1 CL1-2	Students in this grade band may benefit from initial experiences building with the blocks without the time constraint. Students at this grade level may benefit from extended viewing time of Quick Block images or building with the image left on the screen. Conversations about patterns observed or subitizing will be helpful in scaffolding students toward success with this routine.
3–5	AL1-2 AL1-7 AL2-5 AL2-8 BL1-9 BL2-2 BL3-6 CL1-3 CL3-8 CL3-9	Students in this grade band may benefit from limited introductory experiences building with the blocks without the time constraint (allowing the image to stay on the screen during build time). Extended viewing time can be provided on the basis of students' needs, but the image should still flash and disappear once students are familiar with the routine. Mathematical conversations focusing on what they saw and how they saw it can be helpful with the image still showing when teachers are beginning to develop this thinking routine with students. Once students develop the necessary vocabulary, switch to using the standard method.
6–8	AL1-6 AL1-8 AL2-7 AL3-5 BL1-5 BL2-1 BL2-5 CL1-5 CL2-6 CL3-7	Students in this grade band may benefit from initial experiences with level 1 figures before moving on to more advanced figures. Conversation should focus on the building process and on seeing structure in level 1 figures before moving to more complex figures, allowing students to begin developing pattern recognition.
9–12	AL1-8 AL2-7 AL3-4 BL1-10 BL2-4 BL2-9 CL1-3 CL2-3 CL2-10 CL3-6	Students in this grade band may benefit from the challenge of more complex figures after success with level 1 figures. Sufficient time for conversation and student explanation is still required at this grade level to develop spatial sense. Students should be encouraged to use mathematical language and vocabulary when describing their process of building the figures.

Also see the appendix, our top 10 images for each grade band.

translation, to find an alignment between their remembered mental imagery and the physical blocks in front of them. They do this while also attending to the colors of the blocks and the way blocks or groups of blocks relate to one another. This is a challenging task, especially the first few times students attempt it. As the images' level of difficulty increases, students are challenged to consider whether or not their physical creation is accurate and are often seen thinking about how to turn the construction so as to represent the perspective of the given image.

We have used Quick Blocks with students of all ages. Engaging students of different ages requires some thoughtful yet minor modifications on the part of the teacher. In table 1, we share suggested modifications teachers might consider when enacting the Quick Blocks routine with children in various grade bands.

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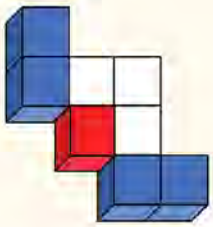
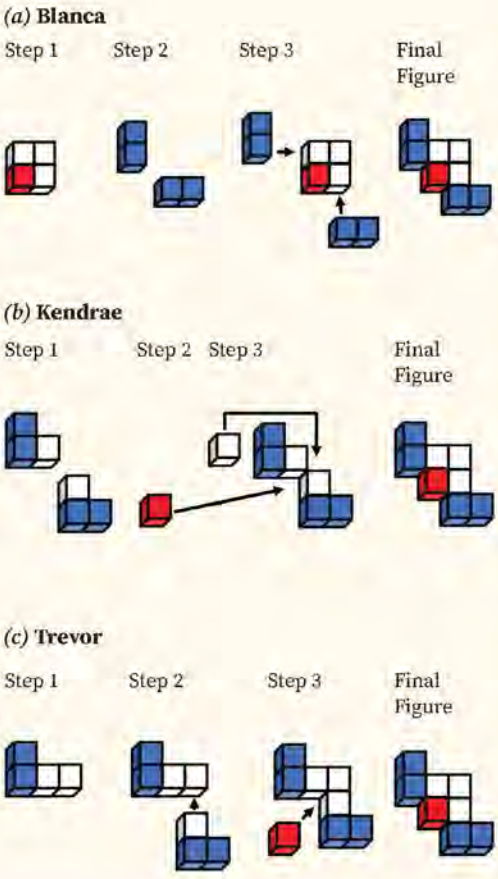
Fig. 8



Original image shown to students

Fig. 9



Demonstration of the steps involved in (a) Blanca's, (b) Kendrae's, and (c) Trevor's thinking

We also share our top 10 favorite images to begin with at each grade band (also see the appendix).

EXPLORING QUICK BLOCKS WITH STUDENTS

The first author, Gabriel Matney, began developing Quick Blocks and ran early trials with students from kindergarten to grade 12. He conducted six trials with small groups of three students from similar grade levels. Children were given a set of 10 Quick Block images. From these trials two things became apparent: (1) Some students had an uncanny ability to connect the image to the construction of the physical object, and (2) all students appeared to improve the more they experienced the activity. For example, one trial had three students: a kindergartner, a second grader, and a third grader. In this trial, the children were all shown 10 Quick Blocks images, one at a time, for three seconds. The image was then hidden, and the students used individual connecting cubes to try to represent the image in physical form. After the children had time to construct the image, they were shown the image for another three seconds and given another opportunity to finalize their construction on the basis of the second look. In nine of the 10 trials, the kindergartner correctly constructed the physical representation the first time. Furthermore, the kindergartner was able to more quickly assemble the blocks for each of the 10 images. Therefore, we note that although students may display a wide variance in ability, success with this task does not depend on age or grade level.

The second-grade student needed two viewings of the image to successfully construct the first six images. Similarly, the third grader needed two viewings of the first seven images to be successful. In other words, the second- and third-grade students were unable to successfully construct the image with blocks after the first viewing until they had done several Quick Block images. As they learned and explored the relationship between the cubes and the images, their ability to orient and assemble the cubes according to the image improved. During the last several years, we have shared Quick Blocks with mathematics teaching colleagues at all levels—elementary, middle, high school, and university preservice teachers. Across these cases, we noticed sets of students who find early success with Quick Blocks much as the kindergartner did. Similarly, much as the second- and third-grade students, all students became more successful as they gained experience with constructing cubic imagery and discussing their visualization of the image with one another.

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Appendix 2 (continued)

One Image, Diverse Ways of Seeing

We share one excerpt of a Quick Blocks dialogue to reveal the diversity of ways students construct images. The excerpt also demonstrates the way students have the opportunity to bring in academic language, question one another, and adapt explanations to help others understand. In the following scenario, these sixth-grade students have been working on a unit involving symmetry, reflections, translations, and rotations. The teacher enacted the routine with the image shown in figure 8, giving the students two opportunities to view and construct the image.

Teacher: [*Motioning for the class to come together.*] Blanca has volunteered to share her way, so let's focus on her ideas at this time.

Blanca: At first I saw a square in the middle with three white and one red cube. Then I quickly saw four blue cubes composed two up and down and two side to side, like this. Then I put all three pieces together. Any questions? (See figure 9a.)

Maggie: I also saw that there were four blue, but today the picture was so fast, so I didn't remember, umm, at first, where the blue cubes went. How did you know where to put the blue blocks?

Blanca: It was fast! But from before, when we talked about negative space, I saw the two Ls like in other pictures. So I knew the blue created those [*two negative space Ls*] by branching out from the sides of the square. Any more questions?

Class: [*Shaking their heads no. Kendrae raises his hand, and the teacher motions to him.*]

Kendrae: I also saw Ls, but I saw different Ls than Blanca. I saw two Ls that are made up of two blues and one white. Then those Ls were held together by one red and one white cube, like this. So that is how I did it. Any questions or comments on my way? (See figure 9b.)

Trevor: I also saw that, but I saw a loner L on top with two blue and two white. So I made both Ls and then snapped them together with the one red in the middle. (See figure 9c.)

Kendrae: I like your way too.

Class: [*Motioning to Kendrae and Trevor that they agree with the method as Tameka raises her hand.*]

Tameka: I noticed that Kendrae's Ls are translations of one another. Well, kind of. You see how you can just slide one on top of the other and they match? But then I noticed that the colors do not match. [*Shrugging*] So, are they a translation?

Teacher: Hmmm, Tameka is posing an interesting question here. Does everyone see the two Ls Kendrae and Tameka are talking about? [*Class nods affirmatively.*] OK, let's spend a moment thinking about Tameka's question with our shoulder partners. Are Kendrae's Ls a translation?

Class: [*Pairs of students begin talking and manipulating their cubes. Several pairs of students bring up different rigid transformations during their discussion. The teachers ask two different pairs of students to share their ideas.*]

Emilio/Daria: We think these are not just a translation. But if we start with the bottom L (two blue cubes with a white on top) then we can take the top L, reflect it like this, then rotate it, and then it matches up where the white cube is on top. See?

Arya/Chris: We kind of agree with Emilio and Daria's idea but we have two answers. If you look at both shape and color, then Emilio and Daria are right: It takes more than a translation to match the Ls. But, actually, if you just consider the shape, like if all the blocks were blue, then the Ls are already oriented the same, and they are the same shape, so a translation would show that they matched.

In this scenario, students demonstrated their different ways of conceptually subitizing (Clements 1999). Whether focusing on color (Blanca) or on self-similar pieces (Kendrae), students quickly recognized the number of cubes needed to construct the image. We often see students choosing to subitize in ways that involve the images' shape, pattern, and color (see also videos 5 and 6). Depending on the image, students discuss both additive and multiplicative ideas related to what is needed to construct the image. One of the reasons we chose to share this excerpt was to highlight one of the many times our students have decided to bring in the academic language of their current unit of study. Tameka's question about Kendrae's Ls allowed the teacher to seize a short moment of connection to the unit, and it allowed the students to explore the mathematics of the image further. Students were able to flexibly consider possible rigid transformations and determine their applicability on the basis of the number of conditions (shape and/or color).

FINAL THOUGHTS

Spatial sense is an important aspect of our daily life and contributes to students' success in other areas of mathematics (Cheng and Mix 2013). Although we have shared

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Appendix 2 (continued)

specific details about kindergarten to grade 6 experiences with Quick Blocks, we have found that middle school, high school, and college students found the activity challenging and enjoyable. In particular, using the activity helped our high school students begin class with something everyone found challenging yet allowed students who struggle with elements of number to continue their development in an accessible way. Number sense and subitizing is one area that can be easily coupled with experiences that develop students' spatial sense. Through tasks such as Quick Blocks, students can further develop such ideas as unitizing, composing and decomposing of number and shape, orientation of objects in space, and mental and kinesthetic manipulation of imagery.

Quick Blocks has been one way in which all students can share their mathematical voice in our classrooms. It has helped our classroom communities develop norms conducive to thoughtful mathematical discourse for learning. We continue to consider and create new variations of our Quick Block activities. We invite the mathematics teaching community to use them with students and provide further thoughts and comments. —

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Appendix 3. Sample Feature Article

Access digital content at nctm.org/mtlt113e1q1

PK 12

A STEAM Exploration of Tiny Homes

Fourth-grade students explore tiny homes as a solution for homelessness in a mathematics-rich STEAM inquiry.

Daniel Edelen, Heather Simpson, and Sarah B. Bush

Creating opportunities for students to learn mathematics in authentic ways is key to positioning them as the problem solvers of tomorrow. Outside the classroom, mathematics does not exist in a vacuum but rather as an integrated part of everyday life. Our work centers on meaningfully engaging students in grade-level mathematics content and practices through authentic science, technology, engineering, the arts, and mathematics (STEAM) inquiries. We believe the incorporation of the *M* in STEAM must extend beyond a tool to address science and engineering standards (Bush, Cox, and Cook 2016; Bush and Cook 2019; NCSM/NCTM 2018). Here, we present an elementary STEAM inquiry in which a diverse Title I classroom of fourth-grade students in the southern United States engaged in solving the issue of homelessness for one family in need.

DESIGNING 3D-PRINTED HOMES FOR A FAMILY IN NEED

This inquiry was developed to capture the power of building tiny homes for those who are homeless. A recent NBC article (https://www.nbcnews.com

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Appendix 3 (continued)

/mach/science/could-3d-printed-houses-help-solve-homelessness-problem-ncna860791) stated that 3D-printed homes could very well be a possible solution to poverty in the United States. We elected to create a mathematics-rich STEAM inquiry centered on homelessness. A central tenet of our work is to create equitable learning opportunities for each and every student where they have access to high cognitive demand experiences. These inquiries should allow for each and every student to enter the problem, use multiple solution strategies, and ultimately relate their own life experiences to the content.

Our inquiry was completed at the beginning of the school year during six one-hour sessions with four of the six days focusing on mathematics. The following portion of the inquiry was completed during the second author's mathematics block; the remaining components were integrated into the science block. Students were tasked to design, prototype, and 3D print a scaled version of a tiny home.

The inquiry was designed to extend students' current understanding of multiplication. Students in this class generally demonstrated a procedural understanding of multiplication. Therefore, we planned this inquiry to foster a stronger conceptual understanding through an authentic context. The mathematical learning goals of this inquiry centered on multiplicative thinking and problem solving as outlined by the Common Core State Standards for Mathematics. The first learning goal was focused on 4.NBT.B.5, "Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations" (NGA Center and CCSSO 2010, p. 29). The second mathematical learning goal was based on the overarching standard of 4.OA.A, "Use the four operations with whole numbers to solve problems" (NGA Center and CCSSO 2010, p. 29). Students were initially placed in design teams of three, which we intentionally created to allow for conversations in which all students would have a voice and be able to discuss their thinking. We began our inquiry with learning about the family. Figure 1 displays the bio card for the Calder family.

In the following sections, we describe our students' mathematical thinking, productive struggles, learning, and, ultimately, the transformative experiences that took place during the culmination of this STEAM inquiry.

Session 1: Building Empathy through Mathematical Discovery

During the first day, students were to make sense of the Calders' financial situation. A critical component of any STEAM inquiry is developing empathy toward the person(s) for whom the problem is designed around (Edelen et al. 2019), helping teachers develop students

Inquiries should allow for each and every student to enter the problem, use multiple solution strategies, and ultimately relate their own life experiences to the content.

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Appendix 3 (continued)

Fig. 1

BIO CARD: THE CALDER FAMILY

The Calders are a family of three. Janice is the mother, and her two children are named Susan, age 10, and Charlie, age 4. They have been homeless for 12 months. The Calders used to live in a two-bedroom apartment, but it was just too expensive. They have been staying at a community shelter and have a very strict routine. Janice wakes up her children at 5:30 a.m. to get them ready for school. Susan begins to pack their clothes after she gets dressed to wait in line for her family's breakfast. That gives her mother time to help Charlie get dressed; he's only 4 and won't start school until next year. After they eat breakfast, they rush to catch the city bus. Susan's school does not have regular bus pickup at the shelter. Once the city bus drops them off, they still have to walk 20 minutes to get to the school. Susan is tired by this time, and has trouble keeping her eyes open during morning lessons. While Susan is at school, her mother and Charlie go around the city trying to find a job. Janice has trouble finding work. She then goes to the laundromat almost daily to wash what few clothes they do have. Janice wants the best for her children. She knows that it is difficult for Susan to focus during school and lonely for Charlie, who doesn't have anywhere to play. The children are both very quiet and are often very sad because they can't play with any other kids after school. They need your help!

The class read the bio card together and then paused so students could imagine themselves in Susan's place.

Fig. 2

Exploring Janice's Last Job

Janice has had trouble finding a job. She recently said, "It has been extremely hard. I just can't find the right job." Your task is to explore some of her expenses to learn more about why the Calder family became homeless.

Step 1: Find the sum of their monthly expenses.

Calder Family Monthly Expenses	
2 Bedroom Apartment Rent	$1,000
Bills (electric, water, sewage, cell phone, etc.)	$300
Grocery	$400
Unexpected Expenses	$100
Total monthly expenses	

Step 2: Janice stated that her last job was not right for her. When she first got the job, she was excited. The job paid $9.00 per hour. In fact, $9.00 is actually above the minimum wage of $8.25 per hour. She said that she needed to look for another job. Let's find out why.

Step 3: Below is Janice's pay per day. She worked 8 hours and made $9.00 per hour, which is equal to $72.00 per day. Find out how much she made per week.

Monday	Tuesday	Wednesday	Thursday	Friday	Total weekly pay
$72.00	$72.00	$72.00	$72.00	$72.00	

Step 4: Estimate 4 weeks in a month. Write in her weekly pay below and find her total pay for all 4 weeks.

Week 1	Week 2	Week 3	Week 4	Total pay per month

Step 5: Why do you think Janice needed another job?

Exploring Janice's Last Job: Learning about expenses versus wages

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who are compassionate, see strong purpose in the mathematics they are doing, and who want to make the world a better place (Bush and Cook 2019). As a class, we read the bio card (see figure 1). Susan was intentionally included to help our students feel connected to the family because they are of a similar age. As students read the bio card, we paused and asked them to imagine what their mornings would be like if they were Susan. Students were prompted to close their eyes and visualize how different their lives might be if they too had to struggle simply to arrive to class each day.

Although bio cards position students to build empathetic thoughts, true empathy is developed from experiences (Rogers 1975; Cooper 2011). Exploring Janice's Last Job (see figure 2) was given to student design teams to begin to think about homelessness as it relates to the Calder family. This inquiry was inspired and adapted from the work of Felton-Koestler, Simic-Muller, and Menéndez (2017).

When students begin to think multiplicatively, they need to see how such thinking connects to addition. Figure 2 provides an authentic opportunity for students to discover the difference between the Calders' monthly expenses and monthly income. Students added $72 iteratively five times to obtain a total weekly pay. We ultimately wanted students to begin thinking about repeated addition as multiplication. We used questions to drive student thinking such as, "How many times did you add $72?"

Students responded, "Five times."

This allowed us to ask, "If you added $72 five times and obtained an answer of $360, I wonder what $5 \times \$72$ would be?" (This was written on a large white board.)

One student exclaimed, "It would be the same! I never knew we were doing multiplication."

As design teams began to reach the conclusion that Janice could not afford to provide for her family with a job that paid only $9 per hour, they were upset. It was clear that our students were deeply enthralled by the inquiry and truly wanted to help a family that they themselves had never met. Developing this level of buy-in through empathy is critical for transformative learning experiences. We then played a video (https://www.youtube.com/watch?v=wCzS2FZoB-I) that demonstrated the process of 3D printing a tiny home. Students learned from the video that a tiny home could be printed for $4,000. We could see our students thinking of the Calders. Students were tasked to determine whether a 3D-printed tiny home with a monthly payment of $400 could change the Calders' situation. Without being prompted, students revisited step 2 of Exploring Janice's Last Job (see figure 2) and determined that Janice could afford a tiny home as compared to the apartment. One design team stated, "Janice would have $240 left over each month!" We then presented students with their problem statement (see figure 3).

Sessions 2 and 3: Multiplication through Scaling

The student design teams began day 2 by revisiting the bio card and problem statement. Once they were clearly focused on the inquiry, students were provided with a blank piece of paper and asked to brainstorm as a team to critically think through planning a blueprint with the following interior house requirements: one kitchen, one bathroom, two bedrooms, one living room, and anything else they thought the Calders might need. During the brainstorming, we introduced the idea of drawing the blueprint to scale. Students were tasked to ensure that their mock-ups were drawn

Fig. 3

Problem Statement: The Calder family needs your help!

We have been hired as a design firm to work on affordable housing with this family in mind. Our task is to design, prototype, and test a tiny home for them to live in. It is important that we include the most important aspects they might need to make their life more comfortable. Remember to keep the Calder family in mind as you design for them. The Calder family lives in the south, so we need to consider warm weather in our plans. The Calders are counting on you and your expertise!

Problem statement for 3D-printed homes inquiry

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Appendix 3 (continued)

Fig. 4

Tiny-Home Measurements in Real Life

- **House:** 36 feet long by 12 feet wide
- **Kitchen:** 10 feet long by 8 feet wide
- **Bedroom:** 10 feet long by 8 feet wide
- **Beds:** 6 feet long by 4 feet wide
- **Living Room:** 10 feet long by 6 feet wide
- **Hallways:** Must be 4 feet wide
- **Doors:** Must be 3 feet in width

Scale: 2 feet = 1 inch

Tiny-home measurements

Fig. 5

Student thinking through her physical model

proportionally (e.g., the toilet cannot be bigger than one of the beds). From previous experiences, we had learned that often students have difficulty drawing house designs and this influences their thinking about what "to scale" means. We began with brainstorming because we wanted students to think about scaling contextually before beginning the next, and larger, task.

When implementing your own STEAM inquiry, it is essential to plan ahead and ensure that the measurements and the scale are appropriate for each and every student.

Figure 4 displays the measurements of the home. When implementing your own STEAM inquiry, it is essential to plan ahead and ensure that the measurements and the scale are appropriate for each and every student. This should be based on students' mathematical understandings and experiences.

Students were provided with large pieces of paper to draw the perimeter of the house. The scale, 2 feet = 1 inch, was presented to students to discuss. Scaling is a difficult concept to grasp because it is naturally abstract. To combat this and provide a concrete manipulative, students were given squares to aid in making sense of the scale. We placed a square under a document camera to display to the class and engaged students in the following conversation:

Teacher: This is one square; the length of the edge of this square represents one inch of our scale. Why would we need to draw our house smaller?
Student: Because we don't have paper big enough!
Teacher: Very good! Think about architects; they have to carry around all their blueprints, sometimes for very large buildings. We are doing the same thing right now. This square's edge represents one inch, so how long is that in real life?
Student: It's two feet!
Teacher: Very nice. So if I wanted to represent 10 feet with my square edges, how many squares would I need? Turn and discuss please. Be sure to use the edges of your squares.

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Fig. 6

Laying out groups of craft sticks to make sense of the problem

This question initiated critical conversations. During this time, we circled around the room and observed students skip counting until they reached 10.

Students began by estimating how many squares' edges (a length measurement) would be required. The skill of estimation is critical in building number sense. Estimating the number of edges was instrumental in students making sense of the scale. Most students lined up squares by skip counting by 2. If they did not have enough squares to reach the desired real-life measurement, they would think about how many more squares they would need. This turned out to be an effective way to build multiplicative number sense. As teachers, we were able to ask questions surrounding the multiplication associated with the square edges such as, "If you have 18 squares edges on a side, and each edge represents 2 feet in real life, how could we represent this as a multiplication sentence?"

The scale was intentionally planned before the lesson to allow students to notice a pattern. Once students began to understand the pattern of the scale as either halving or doubling, they began to measure out the remaining sides of the house. Students completed session 2 with a finished outline of the home. Session 3 consisted of students working to model the remainder of the house on the basis of their early blueprints. Although scaled drawings are not a fourth-grade standard, our goal was not to have our students be proficient in drawing to scale. We wanted to use the context-driven experience to engage students in multiplicative thinking. Through preplanned and scaffolded instruction, students were able to make sense of the mathematics. The scaled drawing portion of the STEAM inquiry also provided an authentic opportunity to interpret patterns in multiplication.

Session 4: Physically Modeling a Tiny Home as Groups of Craft Sticks

Students started the fourth session reflecting on the bio card and problem statement. The major focus of the fourth day was to build to scale a physical model of their home for subsequent scientific testing of home insulators. This inquiry was implemented in warm temperatures in the southern United States. To authentically create an opportunity for insulating the homes, we elected to model the homes with craft sticks. In the latter parts of the inquiry, students used such materials as foil, cotton balls, felt, and construction paper to insulate their model to maintain constant temperatures.

Students placed their models under a heat lamp and collected temperature data to determine the best insulators. See Edelen, Simpson, and Bush (forthcoming) for more information about the science portion of this inquiry. Students were told that each craft stick measured 6 inches in length. Students were tasked to determine how many sticks they would need to re-create the length of the home to scale. We created an opportunity to learn about multiplication with groups of sticks as a concrete example for students to consider. At this point, students were given the real-life height of the tiny home as 12 feet. Once students reached the conclusion that their scaled model would be one craft stick tall, they were prompted to build the house in groups of four craft sticks and to make the sections of the home. Figure 5 shows a student attempting to make sense of the necessary measurements to correctly model the home.

As mentioned above, preplanning the scale was critical to the success of this scaling-centered STEAM inquiry. Figure 6 displays how the groups of four craft sticks were connected to create one side of the tiny home. We asked students to think about how many groups of sticks were on each side and in the entire model. This physical representation of the tiny home created an opportunity for each student in the classroom to make sense of the associated mathematics. One student explained the number of craft sticks on a given side as, "This side is 3×4 because I can count that there are 3 groups. In each group is 4 sticks, so, 3×4 is 12 sticks. I counted them to show I was correct." The groups of craft sticks provided a concrete experience for students.

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Appendix 3 (continued)

Fig. 7

Interpreting Janice Calder's height

The model also created a real example of what it means to multiply a number by 1: Two of the sides had 3 groups of 4, and two of the sides had 1 group of 4 sticks. Students often simply memorize that multiplying a number by 1 always equals the same number. However, the model created a real example for students to think through why this is true. Students reached the conclusion that multiplying by 1 means that there is a single group, in this case, that there is 1 group of 4 sticks. Figure 6 showcases how many students connected their physical models back to the blueprints they drew on the previous days.

After students finished building their models, they were asked how tall the Calder family would be if we were to draw them next to the craft sticks. Students were told that in real life Janice measured 5 feet 6 inches, Susan, 4 feet, and Charlie, 3 feet tall. This was especially challenging for the students because the numbers could not be easily halved; this was especially true for the height of Janice. They first modeled the height of Susan as 2 inches. After each group reached this conclusion, we again brought out the square manipulatives and encouraged the use of estimation.

Students were able to reason that 1 square was equal to 2 feet and 2 squares were equal to 4 feet, so Charlie must be between 1 and 2 squares tall. We encouraged students to cut a square in half to help aid in their thinking. Students reached the conclusion that Charlie would measure 1 1/2 inches in the model. Figuring out Janice's height was the most challenging (see figure 7).

Students noted that Janice's height was more than 2 squares high because Janice was more than 4 feet tall. Students then tried 3 squares but noted that would represent 6 feet. At this point, we prompted students to first focus on the 5-foot portion of her height. Students remarked that 5 feet was between 2 and 3 squares. Students then stacked 2 squares and cut a third square in half (for a height of 1/2 inch to represent 1 foot). Once students had modeled 5 feet with their squares, we displayed this on the document camera. We asked students to think about 6 inches in relation to 1 foot. Students exclaimed that 6 inches was half a foot. We then asked students how we could represent 1/2 foot using our squares. One excited student responded, "We could cut a square in half to show 1 foot and then cut it in half again because 6 inches is half of 1 foot." To consider how tall the "half of a half" piece was, we asked students to think about what 1/2 of a 1/2 was, and we allowed them to measure using their rulers. The use of purposeful tools and their previous

Table 1 STEAM Connections

Subject	Learning Component
Science	• Insulate tiny home to learn about temperature change. • Conduct experiments to determine the best insulation based on collected data.
Technology	• Use Tinker Cad to digitally model a tiny home to scale. • Use a 3D printer to print a scaled-down version of the physical tiny-home designs.
Engineering	• Construct and design explanations of how their insulations meet criteria. • Explain the cause-and-effect relationship of using certain insulators on the tiny home.
Arts	• Design a tiny home to best meet the needs of the Calders, attending to aesthetics and functionality.
Mathematics	• Draw a tiny home to scale. • Use multiplication to make sense of the minimum wage disparity and make conclusions about homelessness based on multiplicative thinking. • Use multiplicative thinking to create physical and software-based prototypes of a tiny home. • Use multiplication to model and solve real-world problems.

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Appendix 3 (continued)

experiences with the scale and the model of the tiny home allowed students to make sense of the heights of the family within the squares (and cut pieces) representation.

STEAM FOR STUDENT LEARNING

This inquiry was designed to extend students' current understanding of multiplication. We used collected student work samples as well as the district summative assessment to assess student learning. Throughout the inquiry, students demonstrated growth in their conceptual understanding of multiplication. After this inquiry was concluded, the classroom teacher was routinely able to make connections back to students' mathematical experiences during the inquiry to help students make sense of multiplication. Although the inquiry took six days to complete, only four days took place during the mathematics block (the other two took place during the science block). We strongly believe that although STEAM inquiries, such as this one, take time to plan and implement, the payoff of providing students with transformative learning experiences is priceless. We have included table 1 as a reference of the STEAM content integration for this inquiry.

CONCLUSION

As students engaged in mathematics throughout this inquiry, we were struck with how dedicated they were to their learning. Students worked to learn, not simply to perform well on an assessment but to truly make someone's life a little better. We concluded that giving students experiences such as these should exist in all classrooms. STEAM education is for each and every student.

"I never really thought math would actually make sense. It's a lot of numbers that aren't really easy to understand, you know. After seeing the different ways to solve the problem and help someone in need, I think I could really do it. I had no idea I was doing multiplication, and I definitely never thought I could before!" a child exclaimed after completing this mathematics-rich STEAM inquiry. Students need opportunities to experience mathematics as a way of making sense of the world. Providing experiences such as these allows students to ground their thinking and build it for the future. It is not enough that students understand mathematics in a vacuum; they also need to see the usefulness of mathematics in making sense of the world. Our hope is that they can grow to one day use this knowledge to make the world better. —

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Appendix 4. Sample Focus Article

Access digital content at nctm.org/mtlt113301fp

PK — 12

Creating Joy in PK–Grade 2 Mathematics Classrooms

Children experience joy in well-designed mathematics classrooms. This article describes five research-based practices for bringing joy into PK–2 mathematics lessons.

Amy Noelle Parks

After more than 30 years in early childhood education, I have developed observational shorthand for recognizing excellent mathematics instruction when I am in schools: I look for joy.

My choice of the word *joy* is intentional because I am not looking for PK–2 mathematics classrooms that are *fun*. As Desmond Tutu and the Dalai Lama explain in *The Book of Joy*, "Being joyful is not just about having more fun. We're talking about a more empathetic, more empowered, even more spiritual state of mind that is totally engaged with the world" (2016, p. 63).

In well-designed mathematics lessons, children can experience many of the characteristics of joy described in the book, including wonder, excitement, pride, and pleasure. So far, little research has been conducted on joy in schools, but psychological work on happiness suggests that children who lose themselves in their work and see their schoolwork as meaningful are both happier and more successful (Csikszentmihalyi 2002). In contrast, a growing body of research has shown that negative emotions such as anxiety interfere with mathematics learning, even for young children (Ramirez et al. 2013).

FIVE STRATEGIES FOR CREATING JOY

If you would like to strengthen your mathematics teaching through joy, here are five research-based strategies to get you started.

1. Create space for play. Play is a powerful tool for reducing stress and for increasing opportunities for mathematical learning (Wager and Parks 2014). Providing free play with a few well-chosen materials—such as counting collections (Schwerdtfeger and Chan 2007), wooden or Lego® blocks (Casey and Bobb 2003), puzzles (Clements, Wilson, and Sarama 2004), or linear board games (Ramani and Seigler 2011)—can give children important opportunities to relax while exploring mathematics. For example, the four-year-old in video 1 began counting his toy bugs without prompting. The teacher enriched this engagement by asking him to restate the total, which encouraged cardinality, and by asking him to count again, which helped to develop one-to-one correspondence. (Both skills are called for in the Common Core content standard KCC.B4: Understand the relationship between numbers and quantities [NGA Center and CCSSO, 2010].) Moments like this demonstrate how even skill practice can be done in the spirit of joy.

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> In well-designed mathematics lessons, children can experience many of the characteristics of joy including wonder, excitement, pride, and pleasure.

2. Allow children to make choices. It is easier to experience joy when we get to make some decisions about how to spend our time, who to spend it with, what materials we can use, and how we hold our bodies. Providing choice may be as simple as allowing children to decide where to sit while working on a problem or as elaborate as providing a choice of three stations to practice the same skill in different ways. For example, children could choose to play a make-a-10 card game with friends, use two colors of Unifix® cubes to make 10 trains on their own, or make a poster of all the ways they can imagine making 10 with two or three numbers with a partner.

3. Offer problems that allow for exploration, social interaction, and material engagements. As Desmond Tutu and the Dalai Lama point out, wonder and pride are sources of joy, and meaningful problems can provide these feelings. Problems can come from whole-class wonderings, from other content areas—"If the giant in *Jack in the Beanstalk* is 10 times as tall as a person, how tall is he?"—or from daily routines, such as counting collections or story problems. Carpenter and colleagues (2017) provide a variety of strategies to joyfully engage PK–2 children in solving story problems, including providing materials for modeling and opportunities to talk with peers.

Children's joy can spring from their pride in solving a challenging problem and from their chance to present their thinking to classmates. Social interactions around problems not only provide joy but also give young children the chance to construct viable arguments and critique the reasoning of others (SMP 3).

4. Relax a little about time on task. In my mathematics methods classes, I often give my students—future elementary school teachers—cooperative problems. Although all groups ultimately solve the problems and have fabulous mathematical conversations, sometimes their talk slides toward other topics, such as the weather or *The Bachelor* or ongoing disagreements from literacy class. When I ask my preservice teacher students how they would feel if I stopped at their tables, snapped my fingers, and ordered them to get back to work, they say that this correction would not lead to joy nor to eagerness to get back to work. The occasional off-task moment will not significantly interfere with

Video 1 Counting Bugs

Watch the full video online

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Children's joy can spring from their pride in solving a challenging problem and from their chance to present their thinking to classmates.

Video 2 Jacket Folding

Watch the full video online

children's ability to learn mathematics (Boaler 2002). For example, in video 2, a kindergartner keeps occupied during a whole-class discussion by folding her jacket (which allows her some opportunities to think about equal parts and symmetry). Her movement does not interrupt the lesson or bother other children. As Horn (2019, p. 32) writes in her fabulous book *Motivated*, overemphasizing time on task forces students "to choose between acting the part of a student, which may feel false, and being their full selves, which may include being playful, silly, or simply interested in the social lives of their peers."

5. Create caring relationships. It is easy to forget that the children we teach are quite young and that they look first to adults for emotional support. Children's warm relationships with their teachers and with one another provide a foundation for taking mathematical risks as well as creating opportunities for joy for both children and teachers. Creating a welcoming environment draws on some classic early childhood teaching practices—greeting children with a smile and by name, taking time to get to know each child, and helping children to name and handle their emotions. Teachers also initiate opportunities for joy when they bring children's home languages, cultures, and experiences into the school because these practices make children feel welcome in the space (Battey et al. 2016; Gholson and Martin 2014). In addition, a classroom climate that allows children to comfortably make mistakes (Horn 2019) and to see one another's mathematical strengths (Wood et al. 2018) is also more likely to create joy.

JOY AS A GUIDING PRINCIPLE

The practices named above are by no means the only ways of creating joy in early childhood classrooms. All year long, teachers can use joy as a criterion for teaching decisions. For instance, teachers might wonder if they should—

- present a few deep tasks or dozens of similar computation problems;
- allow children to talk or require them to work in silence;
- provide a choice of tools or assign one;
- focus on growth over time or achievement against a standard;
- emphasize fluency or speed;
- provide diverse instructional settings or the same instructional routine each day; or
- incorporate art, music, or science into mathematics or maintain strict disciplinary boundaries.

The amazing thing about choosing joy in each of these cases is that the choice also leads to richer and more productive mathematics.

So why not choose joy? —

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Appendix 4 (continued)

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Appendix 5. Sample Ear to the Ground (Department) Article

What about Alfred?

Ear to the Ground features voices from various corners of the mathematics education world.

Marian Dingle

It was a December day, when elementary school students are bursting with energy and on the lookout for surprises that the season brings. I had arranged for a special visit from a guest whom my students idolized—my son, a college freshman. They had heard countless stories of his high school football days and faithfully asked me each Monday about Saturday's game. On this particular day, his friend Alfred, also a student athlete, came with him.

Wearing their college football team regalia, they both wowed my students by answering questions and sharing wisdom. We escorted the young men to lunch in the cafeteria, and I introduced them to my various colleagues. Everywhere, they were met with smiles and warm greetings.

Alfred, who had never been to my school before, kept shaking his head. I became curious whether he appreciated the fandom bestowed on him by my students. Despite my characterizing these two as great academicians, my students were ultimately impressed with their athletic potential. Still, Alfred's reaction had nothing to do with my students. It was about the school itself.

When I finally inquired, his response hit me like a ton of bricks: "*It feels like a prison.*"

Momentarily stunned, I ran through a mental inventory; this was a normal day for us, the children were smiling, the school building was clean, the environment was orderly. It turned out that Alfred was reacting to the right angles of the corridors, the students walking in straight lines, and the compliance in the air. Alfred, a Hurricane Katrina survivor, moved to our state when he was in middle school. He said my school felt like his own elementary school, which was several states away.

Immediately defensive, I asked, "Does my classroom feel like that?" And of course he said no. But what would he say? I have a good relationship with him and my son's other friends. They know and respect what I do. I know that they benefited from having that teacher who looked out for them throughout high school, from whom they could solicit advice, who kept tabs on them long after they left their classroom. I tend to be that teacher in my school.

This got me thinking about our mathematical practices. Does the expectation of compliance, as evidenced by the straight lines that Alfred noted, work against the mathematical practices we promote? Are our students free to notice and wonder why school is set up the way it is? Do we allow them to engage in discourse about who and what we highlight (https://www.mariandingle.com/blog/measures-of-center) in the

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Appendix 5 (continued)

Does the expectation of compliance, as evidenced by the straight lines that Alfred noted, work against the mathematical practices we promote?

classroom? During a three-act task, are students free to reject the question we guide them toward? Are we really teaching them to take risks?

Was Alfred ever allowed to notice patterns about which students are tracked into accelerated programming and then wonder how he could jump tracks? How many students have I seen assigned to academic boot camp for remediation or rewarded with free time as if school is the antithesis of freedom (Bell 2015)?

Alfred is an intelligent young man who remains on the dean's list in college. But he never once considered a career in education. None of my son's friends have. Perhaps the time has come to ask students how our classes feel. Perhaps they can tell us what they need. When I stop to ask these questions, I am left with more than I bargained for. Schools are for education, but who is educating whom? _

REFERENCE

Bell, Monita K. 2015. "Are Your Students Prisoners?" *Teaching Tolerance*. October 19, 2015. https://www.tolerance.org/magazine/are-your-students-prisoners. Accessed June 7, 2019.

Marian Dingle, mdingle@comcast.net, has taught elementary school for 20 years in Georgia and Maryland. As a Heinemann Fellow, she conducts action research about the ways in which cultural identity influences student efficacy, agency, and mathematical outcomes.

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Appendix 6. Sample For the Love of Mathematics (Department) Article

Mathematical Art: Lost in Translation

Traditionally, high school geometry has focused on the study of two- and three-dimensional figures, postulates, and measurements. By connecting geometry, art, and history, we can create opportunities for students to experience the excitement and beauty of mathematics that can help to build on larger ideas of reasoning and proof.

Siddhi Desai and Farshid Safi

Mathematics and art have been beautifully and intentionally interwoven throughout the history of many cultures and societies. Mathematical designs consisting of symmetry, lines, and concentric circles can be commonly found in Indian and Islamic art, architecture, and culture. Traditions such as creating Kolams, or Rangoli, are still commonly practiced throughout India and many parts of Southeast Asia. These beautiful geometric designs were the inspiration for this art piece.

All too often, traditional high school geometry instruction strictly adheres to the study of two- and three-dimensional figures, postulates, and measurements (NCTM 2018). This presentation of mathematics results in students feeling a disconnect between themselves and the relevancy of mathematics. One way to engage students and make the beauty and wonder of mathematics more directly evident as well as connected to learning experiences involves enriching educational experiences that link and extend content strands with multidisciplinary domains. When we embrace a multicultural understanding of mathematics, we see how modern mathematics has evolved from the contributions of many cultures across the planet and throughout time (Barta, Eglash, and Barkley 2014). By integrating geometry, art, and history, we can create opportunities for students to experience the excitement and beauty of mathematics that build on foundational geometric ideas and elevate student interest in mathematics and related subjects. —

For the Love of Mathematics, driven by reader suggestions and submissions, offers visual, engaging, and inspiring material for you, the teacher.

REFERENCES

Barta, Jim, Ron Eglash, and Cathy Barkley. 2014. *Math Is a Verb: Activities and Lessons from Cultures around the World.* Reston, VA: National Council of Teachers of Mathematics.

Lappan, Glenda, Margaret Schwan Smith, and Elizabeth Jones, eds. 2012. *Rich and Engaging Mathematical Tasks: Grades 5–9.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). 2018. *Catalyzing Change in High School Mathematics: Initiating Critical Conversations.* Reston, VA: NCTM.

Siddhi Desai, siddhi.desai@knights.ucf.edu, is a doctoral student at the University of Central Florida and a high school mathematics teacher at Oviedo High School in Oviedo, Florida. She is interested in studying the integration of mathematics, art, history, and modeling to engage students in rich mathematical experiences.

Farshid Safi, farshid.safi@ucf.edu, is a K–12 Mathematics Education Faculty member at the School of Teacher Education at the University of Central Florida. He focuses on developing teachers' conceptual understanding of PK–16 mathematics through an emphasis on connections and coherence while engaging and empowering students through mathematical experiences.

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Appendix 7. Sample Growing Problem Solvers (Department) Article

Access digital content at
http://pubs.nctm.org/mt113-01gps

GPS: From Folding to Dynamic Geometry Environments

Growing Problem Solvers provides four original, related, classroom-ready mathematical tasks, one for each grade band. Together, these tasks illustrate the trajectory of learners' growth as problem solvers across their years of school mathematics.

S. Asli Özgün-Koca and Matt Enlow

This month's Growing Problem Solvers department focuses on supporting students' understanding of congruence via the use of rigid motions and transformations.

According to NCTM's *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006), initial understandings of congruence and symmetry begin in first grade as students work with different shapes, recognize them from different perspectives, and reflect on how they are similar and different.

The **PK–2 grade band** task includes an activity asking students to partition a rectangle into two, three, and four equal parts and reflect on equal partitions. Teachers should pay attention to whether students' folds created equal parts. Provide at least 10 cut-out wax-paper rectangles per student, allowing students to use a new piece of wax paper for each different fold so they can keep records of their previous folds. If wax paper is unavailable, lightweight copy paper can be used. This task can be extended by using a variety of sizes of rectangles.

The **3–5 grade band** task continues the use of wax paper and folding, this time focusing on the symmetry lines of different shapes. Students are asked to create and recognize a line of symmetry which splits a shape into two congruent parts that are reflections of each other. Teachers should pay attention to whether students' folds divide the figure into two mirror-image halves.

Students are also asked to identify a line that divides each shape into two equal parts that is *not* a line of symmetry. Because multiple correct solutions exist, the tasks support dialogue among students, especially as students explain their reasoning about how they know they have identified all the lines of symmetry of different polygons.

In the **6–8 grade band** task, students use GeoGebra activities to apply specific transformations to a shape to map it onto a given image. Teachers can extend the conversation by asking how many different correct sequences of transformations there might be, or how one might decide that one solution is "better" than another. Students will most likely assert that shorter sequences are better (although push-back on that assertion might be interesting: "Why is that?"), but students might also offer other criteria.

In the **9–12 grade band** task, students are asked to determine whether the shaded triangle can be mapped onto several other triangles using some combination of the two given transformations. (*Teacher note*: Triangles I and III are possible, but II and IV are not.) Students are encouraged to consider whether multiple sequences of transformations could accomplish the task. In the case of triangles II and IV, students may come to a rapid consensus that it is impossible (using only the given transformations). The teacher may need to play the part of the skeptic: "How do you *know* it's not possible? I'm not convinced." The teacher should push students to try to convince themselves and the rest of the class that it is not possible.

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Appendix 7 (continued)

PK 12

Give students 10 rectangles cut out from wax paper. Students should work in pairs or small groups. Tell students to fold one of their rectangles into two equal parts. Ask them to explain how they know that the rectangle is partitioned into two equal parts.

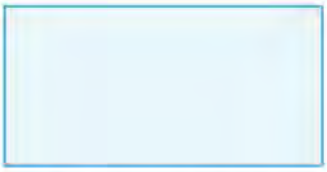
Have students pick up another rectangle and fold it again to create two equal parts in a different way. Continue to have students fold to create two equal parts in as many different ways as possible.

Have students check the partitions that their group members created and respond to these questions:

- Are the two parts equal?
- Do the parts fit together to form an entire rectangle?

Have students fold another copy of the rectangle into four equal parts. Ask them to explain how they know they created four equal parts. Direct students to pick up another rectangle and fold again to create four equal parts. Instruct students to continue to fold to create four equal parts in as many different ways as possible. Students should again review and discuss group members' folds.

Finally, challenge students to fold a rectangle into three equal



parts. Ask students how they know they created three equal parts. Have students pick up another rectangle and fold again to create three equal parts. Instruct students to do this in as many different ways as possible. Students should again review and discuss their group members' folds.

PK 12

Give students multiple wax-paper rectangles, pentagons, and hexagons like those at right. For more challenge, add wax-paper triangles, squares, parallelograms, and rhombi to the set of wax-paper polygons. Then ask students to fold each polygon into two equal parts. Ask students the following questions to answer either individually or in small groups:

- How do you know you have created two equal parts? Do those parts compose the whole polygon?
- With ink or colored pencil, draw line segments on the wax paper where the creases divide each polygon into two equal parts. How do your line segments compare with your group members' line segments? Does each shape have more than one possible line segment?
- How many lines of symmetry does each polygon have?
- Can you fold each polygon into two equal parts in a way that the crease/fold is *not* a line of symmetry? If so, how? Discuss with your group the similarities and differences between a line of symmetry and some other line that splits that shape into two equal parts.

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Matt Enlow, matt.enlow@danahall.org, teaches upper-school mathematics at the Dana Hall School in Wellesley, Massachusetts. He is interested both in the art of problem-posing and in mathematical art. He tweets math-y things at https://twitter.com/cmonmattthink.

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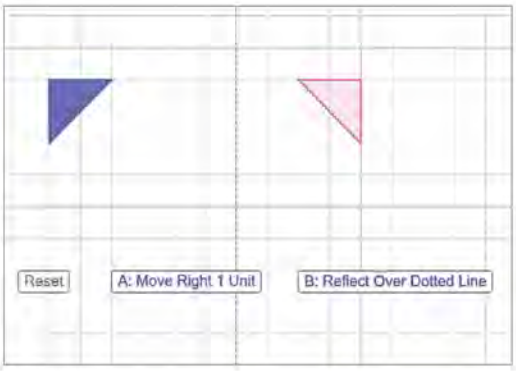
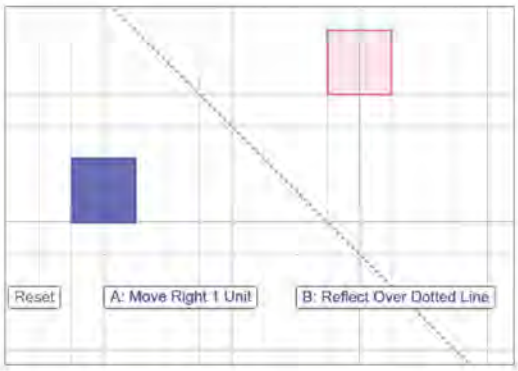
Appendix 7 (continued)

Go to https://www.geogebra.org/m/tv4yhayn and https://www.geogebra.org/m/tbsqshkf to complete the tasks in GeoGebra.

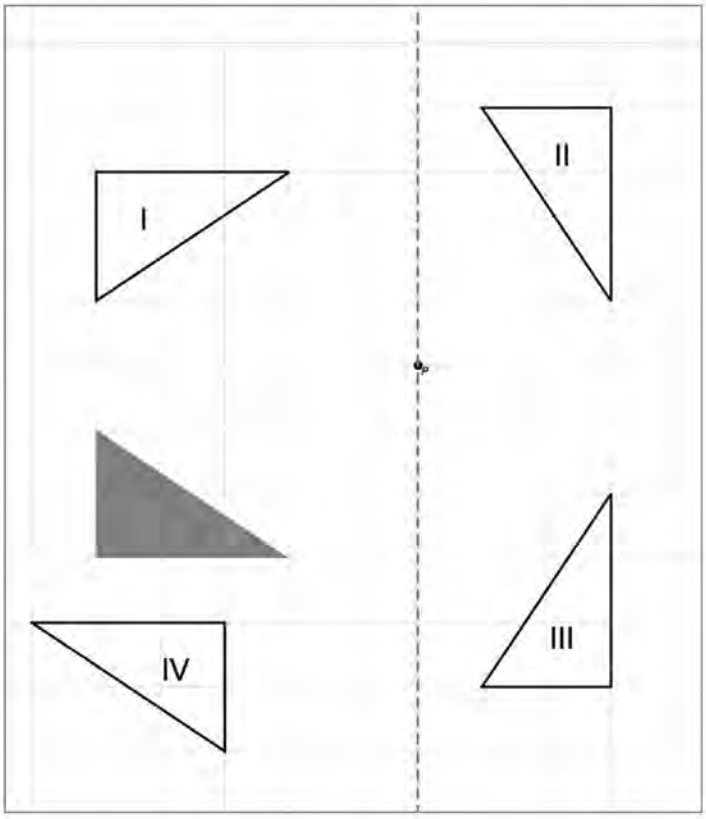




For this task, only two transformations can be used:

A. Clockwise rotation by 90° about point *P*
B. Reflection over the dotted line

1. For each of the four triangles I, II, III, and IV, can you find a sequence of transformations (using only A and B) that maps the shaded triangle onto it?
2. If so, what sequence of transformations accomplishes the task? Compare your sequence with that of a classmate. Are they the same? If not, determine whether it is also correct.
3. If not, how do you know? How confident are you that no such sequence exists? Why?



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Appendix 8. Sample Problems to Ponder (Department) Article

Answer key available at nctm.org/mtlt113012p2p

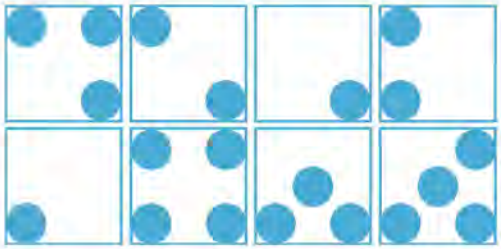
Problems to Ponder

Problems to Ponder provides 28 varied, classroom-ready mathematics problems that collectively span PK–12, arranged in order of grade level. Answers to the problems are available online.

Steve Ingrassia and Molly Rawding

1

Find the numbers that match.



2

Using the shapes, create a pattern. You can add color to the shapes.

3

Hypatia rolled three dice and counted all of the dots. She counted 10 dots. She did not roll any ones or sixes. What three combinations of numbers might she have rolled?

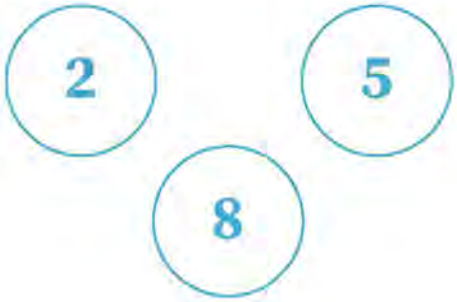
4

On Saturday, Brooks was at his home. Brooks walked 2 blocks north to Max's house. Then, Brooks walked 5 blocks west to Sophie's apartment. Then he walked back to Max's house and then back home.

If Brooks stayed on the sidewalk and did not take any shortcuts or detours, how many blocks did Brooks walk?

5

Using the numbers on the board as many times as you want, what sums can be made between 6 and 23? What sum can be made in exactly five ways? (For this problem, we will consider 5 + 2 + 5 = 12 and 5 + 5 + 2 = 12 to be the same.)



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Appendix 8 (continued)

6

Two numbers added together sum to 20. One number is 10 greater than the other. What are the two numbers?

7

Maryam has 15 flowers. Some are red, some are yellow, and some are purple. If she has more yellow than red, and more purple than yellow, how many of each color might she have if the number of each color is odd?

For further thought: Could they all be even? Why or why not?

8

Using three different single-digit numbers, how can you make this statement true?

____ + 47 + ____ = 52 + ____

9

Continue this pattern in different ways.

Ant, Bear, Camel . . .

10

Ada picked three polygons out of a bag. She counted all the sides of the shapes. Ada counted 12 sides. What shapes might she have picked out of the bag?

11

Owen's birthday cake is a rectangle 12 inches long and 9 inches wide. It is frosted on the top and on all four sides. Owen cuts the cake into 3-inch squares. What is the number of pieces frosted on three sides and the number of pieces frosted on one side?

12

The sum of three prime numbers is a square number. What might the number be?

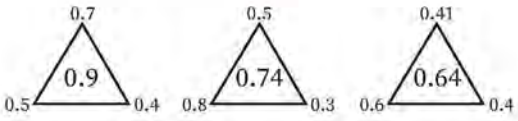
For further thought: Can you find any square numbers that are *not* the sum of three primes?

13

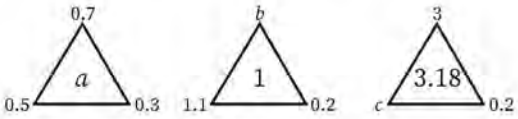
What is staying the same? What is changing? Sketch the next two cases. How many stars appear in case 11?

14

These triangles follow a rule.



What number replaces each variable?



Steve Ingrassia, steve.ingrassia@gmail.com, is a retired high school mathematics and computer science teacher in Chesterland, Ohio. He is interested in applying multiple techniques to solving cross-disciplinary, real-world problems.

Molly Rawding, mollyrawding@gmail.com, is a mathematics specialist/coach at Fiske Elementary School in Lexington, Massachusetts. She enjoys collaborating with teachers to develop students' mathematical understandings through visuals and puzzles.

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Appendix 8 (continued)

15

A drawer contains 5 black, 5 red, 5 blue, and 5 white socks. If you remove socks at random from the drawer, what is the minimum number of socks you must remove to be certain you have a pair that match?

For further thought: How does this change if you start out with 10 of each color? How does this change if you add green socks, so that there are 10 socks of each of 5 colors?

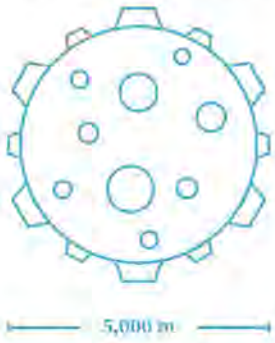
16

The first two terms in a Fibonacci-like sequence are specified. Each subsequent term is the sum of the previous two terms. One example (the Fibonacci sequence) is 1, 1, 2, 3, 5, 8, 13, ... Come up with another Fibonacci-like sequence whose fifth term also has a value of 5.

For further thought: How many different Fibonacci-like sequences are there whose fifth term has a value of 5?

17

A rope encircles the equator of a spherical asteroid with a diameter of 5,000 meters. How long is the rope?



18

If I drive for 3 hours at 40 miles per hour, 2 hours at 50 mph, and 3 hours at 60 mph, what is my average speed?

19

Alice and Belle are born on the same date, but in different years. One year Alice is exactly 4 times as old as Belle. Exactly one year later she is 3 times as old. How old was Alice when Belle was born?

20

A circular pizza with a 16-inch diameter is cut into 8 equal slices. A square pizza is cut into 9 equal-size pieces. To the nearest inch, find the dimensions of the square pizza so that each of its 9 pieces has the same area as one slice of the round pizza.

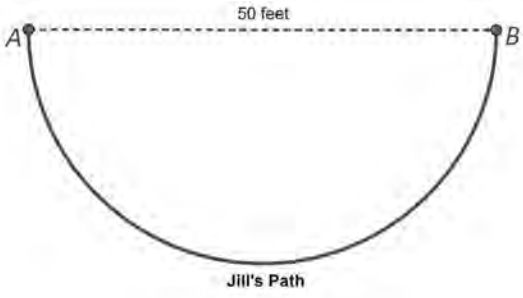
For further thought: If a box is made to exactly fit the square pizza, will the circular pizza fit into the box?

21

Three merchants originally sell an item at the same price. Over the course of several weeks, Merchant A raises the sales price by 10 percent, raises it by another 10 percent, and then lowers it by 20 percent. Merchant B raises the price by 20 percent, lowers it by 10 percent, and then lowers it by another 10 percent. Merchant C raises the price by 20 percent, and then lowers it by 20 percent. Order the merchants' final prices from lowest to highest.

22

Jack walked up and down a hill. The hill's shape is a cone frustrum (a cone with the tip cut off) whose sides make a 30-degree angle with the ground, with a vertical height of 10 feet. Jill went around the hill, which has a circular base with a diameter of 50 feet. Who walked farther?



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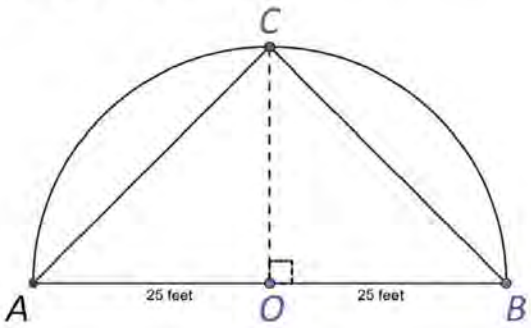
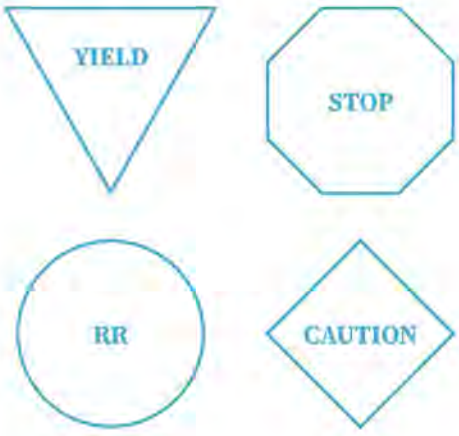
23

Newton is a dog who is attached to a 25-foot-long leash that is attached at point O as shown in the diagram below. He runs in a semicircular arc from point A to point B and back again to point A. Gauss is a dog who is unleashed. She runs in straight lines from point A to point C, point B, and back to point A. If both dogs run at a speed of 10 feet per second, which one returns to its starting point first?



24

Four road signs, as shown below, are exactly the same height. The YIELD sign is an equilateral triangle, the STOP sign is a regular octagon, the RR sign is a circle, and the CAUTION sign is a square. Order the following road signs from least area to most area.



25

The cost of a 30-second Super Bowl television advertisement was \$3 million in 2011, and it was \$4.5 million in 2015. Estimate the cost in 2017 if the costs increase linearly, and if the costs instead increase exponentially.

26

Start with a temperature measured in degrees Fahrenheit and then add 50°F to it. Convert both starting and ending temperatures to degrees Celsius. What temperature must you start with so that the second Celsius temperature is twice the first?

27

A meteorologist predicts a 40 percent chance of rain each day in the next week (7 days). If the meteorologist is perfectly accurate, what is the probability that there will be rain on at least one day during the week?

28

Planet X and Planet Y have circular orbits around their sun, with radii of 150,000,000 and 225,000,000 kilometers, and orbital periods of 350 and 643 days, respectively. Consider a time when both planets are at their closest distance to each other. How many days until they are again at their closest distance?

For further thought: How many days until Planets X and Y are at opposition (in a direct line, but on opposite sides of their sun)?

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Appendix 9. *MTLT* Frequently Asked Questions

Do I need to submit my digital assets at the same time as my manuscript?
Yes. Digital assets can be submitted as supplementary files. If the digital asset reveals the identity of the author, it should be submitted as a "supplementary file not for review."

Does every article require a digital asset?
No. At this time, we do not require the inclusion of a digital asset. However, digital assets are strongly encouraged.

Where can I find examples of digital-first articles?
Appendix 1 in this Author Toolkit contains examples of articles taken from the first volume of *MTLT* that showcase a variety of digital assets.

How long should my video recording be? Audio?
Typically, videos should be no longer than two or three minutes; otherwise you risk losing the attention of viewers. If you have longer videos, consider breaking the video into several smaller videos. Technical specifications for videos and other digital media can be found in appendix 10.

What permissions do I need? (e.g. multimedia, parents/students, other)
Any time you capture video, audio, or written work of individuals, you need written permission. This is true for adults and children (parental permission needed for minors). Permission forms are available in appendixes 11 and 12.

How do I know when I have achieved a digital-first submission?
Digital-first submissions contain digital assets that help the reader be immersed in your content. Examples of various digital assets can be found in appendix 1.

I do not have experience with digital-first publications. How do I work with *MTLT* to write?
The editor-in-chief is always happy to speak with potential authors and brainstorm ideas regarding how to use digital assets in preparing a manuscript. In addition, as a manuscript moves through the revision process, the associate editor will make recommendations for the inclusion of digital assets.

The associate editor has asked that I include a digital component with my revised manuscript. Is there financial support available for this?
No, this is not a service that NCTM can provide. We suggest conferring with your IT professionals or tech-savvy colleagues for help.

Which companies have contracts with NCTM as good resources regarding digital assets for my manuscript?
There are no companies that have contracts with NCTM for the purpose of developing digital assets. However, thus far, tech groups, such as Desmos and GeoGebra, have been willing to work with authors to feature their apps in *MTLT*.

How will my digital assets be referenced in my manuscript?
You should refer to your digital assets in the manuscript just as you would refer to a table or a figure. Statements such as, "See video 1 for a classroom excerpt," are best.

Appendix 9 (continued)

If I have a video, should I also include the dialogue from the video in the manuscript? I am thinking that some readers may not want to watch the video.
No. As the author, you should write your manuscript assuming that the reader watches the video. This assumption aligns with the digital-first philosophy of *MTLT*.

What multimedia sources are off limits?
We encourage creativity and innovative approaches to the use of appropriate multimedia.

Can I link to my YouTube channel (or other media source like this)?
In general, we would prefer to house all digital assets associated with *MTLT* articles on the journal's platform. We recognize, though, that this is not always possible. In these instances, authors are encouraged to work with the editor-in-chief.

I have an idea for a technology, but I do not know how to implement it. How do I get help?
We suggest conferring with your IT professionals or tech-savvy colleagues for help.

Appendix 10. Digital Assets and Figures: Instructions and Technical Specs

Video files

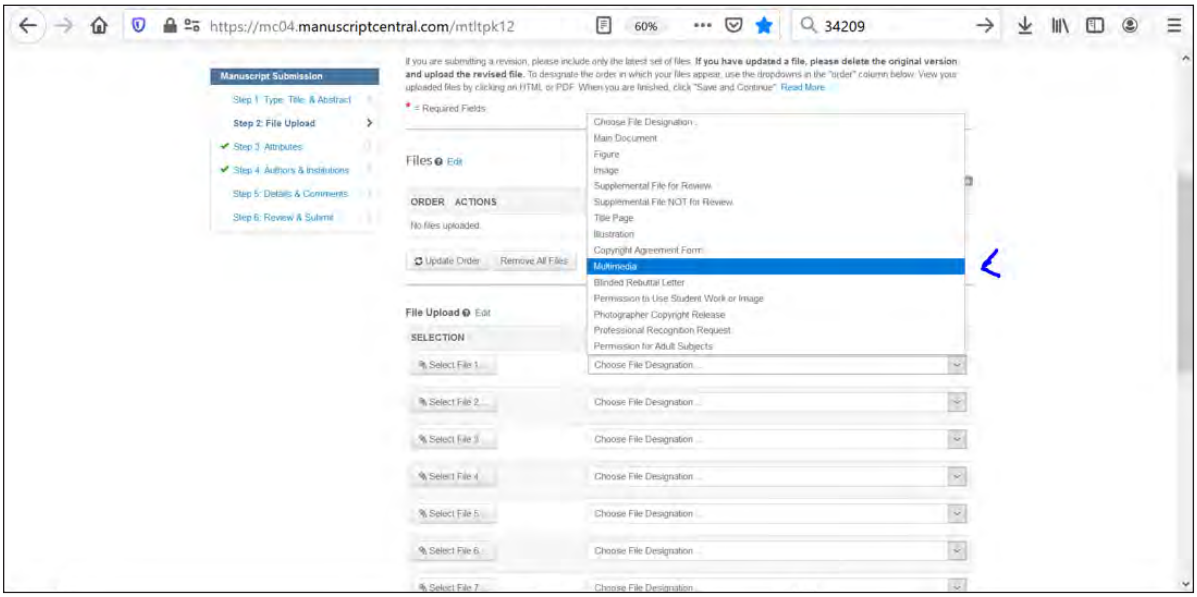
As a digital-first journal, *MTLT* encourages authors to include video content that enhances the message in their manuscript. All multimedia content should be cited within the main text of the manuscript. In addition, a still image (and image description) from a relevant part of the video is needed for use in print.

Specifications for video files:

- Duration: four minutes or less
- Formats accepted: MOV, MPG, AVI, FLV, F4V, MP4, M4V, ASF, WMV, VOB, MOD, 3GP, MKV, DIVX, XVID, WEBM
- Preferred file size: No more than 100 MB

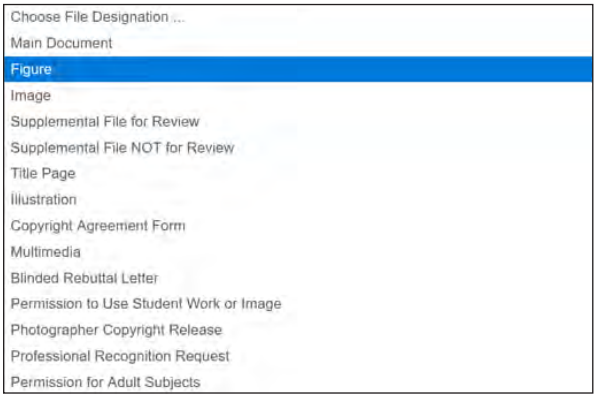
Important! Permission to publish a video is required from all participants who are featured; to ensure that all legal requirements are met, the NCTM permission form (appendix 11) should be used when obtaining adult particpants' signatures. For student participants who are under 18 years old, a parent/legal guardian permission form (appendix 12) is required instead. The signed permission releases should be uploaded into ScholarOne and the file type labeled as "Supplemental file not for review" during submission of the original manuscript. Permission to reproduce is also required from videographers and photographers involved in a video's creation.

When submitting to ScholarOne, during **Step 2: File Upload**, select "*Multimedia*," as shown here, for audio, video, or other digital-only file:



Appendix 10 (continued)

The still image from the video should be embedded in your manuscript file and have a legend accompanying it. If your paper is accepted, you will be asked to provide the still image as a separate file.



When creating a still image, here are a few pointers:

- Try to capture a still image that gets the point of your video across. For example, if you are showcasing use of an app with PK–2 students, take a still image of the output from that app or capture an image of a child using the app.
- Avoid using a personal headshot for your still image.
- The legend explaining the image can be provided within the manuscript text file, placed below the still image.
- After Acceptance: Please label the image file in a way that links it to the video—e.g., "Still_Image_Video_1.eps"

YouTube Videos

Videos featured on YouTube are welcome. For self-authored videos on an author's own channel, the journal strongly prefers that the video source file be provided during submission, rather than as a link to the YouTube channel in the manuscript file. Authors can retain copyright on their YouTube videos if they specify this during submission; the source video file is generally required. If the video is from another channel, authors may instead provide the YouTube link within the text of their manuscript file.

GeoGebra

If the GeoGebra is self-authored, please provide the original, raw .ggb file. If sourced from another user, please provide the correct link.

Desmos

Please provide the original Desmos link.

Geometer's Sketchpad

Please provide the raw file.

Appendix 10 (continued)

Other Apps *

(Such as PowerPoint, LiveScribe, simulations, and so on) Please provide the source file(s) when you upload your manuscript to ScholarOne.

Images and Figures

At acceptance, images/figures must be provided in an acceptable format (see Artwork Guidelines on the next page for more information) at a resolution of *at least* 300 dpi and a width of *at least* 3 inches.

* *A note about large file sizes: the ScholarOne system accepts files up to 100 MB in size. Should your multimedia or figure files exceed this limit, please contact the* editorial office *for additional instructions.*

An Author's Guide for Submitting Artwork

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

This brief overview highlights the main points to be aware of when submitting final production-quality artwork. When preparing figures, refer to printed copies of NCTM publications to get a sense of general size and style. The quality of the reproductions in your book can never be better than the original material you submit to us, so providing the best quality imagery is imperative.
If we judge a piece of artwork to be substandard for printing, we will ask you to provide a suitable replacement or to eliminate the image entirely.

BEFORE SUBMITTING YOUR DIGITAL ART

DO check the resolution of your files to be certain they meet NCTM requirements.
DO submit labeled printouts of all files that correspond to their respective placement in layout. You ***may*** embed art in the Word document, but you ***must*** also provide a JPEG, BMP, TIFF, or EPS file.

MISTAKES TO AVOID

DO NOT assume that a file that looks good on a computer screen is acceptable for print reproduction.
DO NOT submit digital images as PowerPoint files.
DO NOT edit or re-save JPEG files (see "A NOTE ON JPEGS" sidebar).
DO NOT enlarge substandard files (see "A NOTE ON RESOLUTION" sidebar).

RESOLUTION REQUIREMENTS FOR DIGITAL ART

Digital art renders images as a finite number of pixels (ppi), or dots, per inch. The resolution of a file, measured in ppi, limits the size at which a piece of digital art can be reproduced.

BASIC REQUIREMENTS FOR DIGITAL ART

Continuous-Tone Images – 300 Pixels per Inch

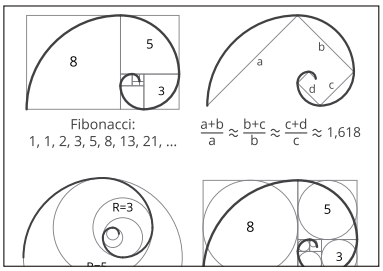
An example of a continuous-tone (raster) image

In continuous-tone images, or raster art, each pixel can vary in color and tonality; transitions from light to dark appear smooth and realistic.

To ensure quality reproduction of continuous-tone images, files must have a resolution of at least 300 ppi. For example, a continuous-tone image with dimensions of 900 × 1500 pixels can be reproduced no larger than 3" × 5".

$$\frac{900\ pixels}{300\ ppi} \times \frac{1500\ pixels}{300\ ppi} = 3" \times 5"$$

Bitonal Images – 1200 Pixels per Inch



An example of a bitonal (vector) image

In bitonal images, or **vector art,** each pixel will be one of two values: 100 percent black or 100 percent white. Such pieces of art require higher resolution to ensure quality reproduction.

The file must have a resolution of at least 1200 ppi. For example, a bitonal scan with dimensions of 3600 × 6000 pixels can be reproduced no larger than 3" × 5".

$$\frac{3600\ pixels}{1200\ ppi} \times \frac{6000\ pixels}{1200\ ppi} = 3" \times 5"$$

HOW TO CHECK FILE RESOLUTION ON A PC

Right-Click on the file and select "Properties" from the menu. Click the "Summary" tab at the top of the "Properties" dialog box. Click the "Advanced" button in the Summary window to display the width and height of the file in pixels.

HOW TO CHECK FILE RESOLUTION ON A MAC

Press the "Control" key, click on the file, and select "Get Info" from the drop-down menu. An "Info" dialog box will appear, and the dimensions of the file will be listed under the "More Info" section of the dialog box.

A NOTE ON JPEGS

JPEG files compress their data to achieve a smaller, more portable file size. This compression is accomplished by discarding some of the data that comprises the image. Each time a JPEG is opened and re-saved in the JPEG file format, the image deteriorates. If you acquire an image from a library, museum, or stock photo agency, request EPS or TIFF format. If JPEGs are the only file format available, do not edit or re-save the image before submitting it. Still, when dealing with JPEGs, do make a copy of the original file as a back-up.

To ensure image integrity, never re-save a JPEG file. If you need to rename a JPEG, right-click on the file and select "Rename" from your menu options. DO NOT rename a JPEG file by opening it and using the "Save As" option.

A NOTE ON RESOLUTION

Some computer programs will allow you to artificially add resolution to a digital file. This will not improve the quality of the image. On the contrary, the image will become fuzzy and pixilated. If you find yourself tempted to enlarge a file, it is a sign that the file is substandard and should not be used.

A NOTE ON IMAGES FOUND ONLINE

Authors must secure rights for any imagery found on the internet that is intended for use in the book.

Computer screens display only 72 pixels per inch. Most images on the web are sized accordingly and are unacceptable for print publication. An image measuring 216 pixels × 360 pixels may appear to be 3" × 5" at 72 ppi on your computer screen, but at the 300 ppi standard required for print reproduction, the maximum size of the image is only about 3/4" × 1".

Information is from Chicago University Press. https://www.press.uchicago.edu/infoServices/artdigest.html

Appendix 11. Figure/Multimedia Permission Form–Adult Participants

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

RELEASE FORM FOR VIDEO AND/OR PHOTOGRAPHS

I, _____ (print name), hereby give my permission to be photographed, videotaped, and/or audio taped. I also authorize these photographs, videos, audio recordings, and/or work samples to be used in all formats and methods of delivery by the National Council of Teachers of Mathematics (NCTM).

I hereby grant permission to the rights of my image, likeness, and sound of my voice as recorded on audio or video tape without payment, attribution, or any other consideration. I understand that my image may be edited, copied, exhibited, published or distributed and waive the right to inspect or approve the finished product wherein my likeness appears.

Additionally, I waive any right to any and all compensation arising or related to the use of my image or recording. I also understand that this material may be used in diverse educational settings within an unrestricted geographic area.

I hereby release NCTM, and any of its associates or affiliates, their directors, officers, agents, employees and customers from all claims of every kind on account of such use.

Full Name(s): _____

Address: _____
_____
_____
_____

Signature(s): _____

Date: _____

Appendix 12. Figure/Multimedia Parental Permission Form–Participants <18 Years of Age

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

RELEASE FORM FOR USE OF STUDENT'S IMAGE OR WRITTEN WORK

Permission is necessary when a student can be identified in photograph(s) or in a video or audio recording, or if their name appears on their written work. Similar forms used by student's school or author's institution may be substituted for form below.

I give my permission for _____'s image to be photographed, videotaped, audiotaped. I also authorize these photographs, videos, audio records, and/or work samples to be used in print, and electronic, and web-based media by the National Council of Teachers of Mathematics (NCTM).

I hereby release NCTM, and any of its associates or affiliates, their directors, officers, agents, employees, and customers from all claims of every kind on account of such use.

Author: _____

Name: _____
(Parent or Guardian)

Home address: _____

_____

Signature: _____

Date:_____

1906 ASSOCIATION DRIVE
RESTON, VA 20191-1502
TEL: (703) 620-9840
FAX: (703) 476-2970
WWW.NCTM.ORG

Appendix 13. *MTLT* Reference Style Examples

Book
Shute, William, William Shirk, and George Porter. 1957. *Solid Geometry.* New York: American Book.

Electronic Book
Shaughnessy, J. Michael, Beth Chance, and Henry Kranendonk. 2009. *Focus in High School Mathematics: Reasoning and Sense Making in Statistics and Probability.* Reston, VA: National Council of Teachers of Mathematics. PDF e-book.

For a downloaded e-book, indicate format as the last part of the citation. For a book consulted online, indicate the URL or the DOI as the last part of the citation. For a freely available electronic edition of an older work, include the URL as the last element.

Chapter in a Book
Oakes, Jeannie. 1995. "Opportunity to Learn: Can Standards-Based Reform Be Equity-Based Reform?" In *Seventy-Five Years of Progress: Prospects for School Mathematics,* edited by Iris M. Carl, pp. 78–98. Reston, VA: National Council of Teachers of Mathematics.

PhD Dissertation
Akdag, Fusun Semiha. 1985. "The Effects of Computer Programming on Young Children's Learning." PhD diss., The Ohio State University.

McGalliard, William A. 1982. "Selected Factors in the Conceptual Systems of Geometry Teachers." Doctoral diss., University of Georgia. Dissertation Abstracts International 44 (1983): 1364A.

Paper Presented at a Conference
Lehrer, Richard, and Paul Smith. 1986. "Logo Learning: Is More Better?" Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, April 16–20, 1986.

Journal Article
Craine, Timothy, and Rheta Rubenstein. 1993. "A Quadrilateral Hierarchy to Facilitate Learning in Geometry." *Mathematics Teacher* 86, no. 1 (January): 30–36.

Electronic Journal Article
Battista, Michael T. 2002. "Building Properly Structured Mental Models for Reasoning about Volume." *ON-Math: Online Journal of School Mathematics* 1 (Winter). https://www.semanticscholar.org/paper/Building-Properly-Structured-Mental-Models-for-Battista/54f8fbd68c2f1dd15b40db52e2d9e5e4926b7754.

Newspaper Article
Braun, R. J. 1993. "Schools That Flunk." *Star-Ledger* Special Report series. Newark (NJ) *Star-Ledger*, October 24–31, 1993.

Tyler, Marshall. 1985. "200th Birthday." *Los Angeles Times.* March 15, 1985, sec. 1A, p. 3.